THE DESIGN CONSTRUCTION THEORY AND PERFORMANCE OF A FRACTIONAL HORSE POWER CAPACITOR START SINGLE PHASE SQUIRREL CAGE INDUCTION MOTOR

by

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P R E F A C E

The details of the design, construction and the performance of a Fractional horse Power Single Phase capacitor start and Induction run motor herein reported constitute a project work undertaken in partial fulfilment of the requerements for the M.Sc. Degree in Electrical Machine Design of the University of Madras. The course for the M.Sc. Degree conducted at the College of Engineering, Guindy, stipulated the submission of an acceptable project work within a specified time and in accordance with the regulation this report is presented. The study was started in May 1958 and completed by October 1958.

Chapter I deals with the scope of the Fractional Horse Power Single Phase Inductbn Motor. The History of the Induction Motor forms the subject of Chapter II. Two of the well known theories such as Rotating Magnetic field theory and the cross field theory are discussed in chapters III and IV. Chapter V deals with the correlation of the cross field and Revolving field concepts. Application of symmetrical components to single phase Induction Motor is shown in Chapter VI. Some of the important problems of the Induction motor such as Noise, Power Losses, Temperature-rise and Rating of Induction Motor have been dealt in chapters VII, VII, IX and X. Chapters XI, XII, and XIII deal with Design, Construction and Performance.

The project work was done under the general direction and guidance of Prof. A. SRINIVASAN, B.E., M.Sc., M.S. (Illinois), Mem. A.I.E.E., Mem. C.I.G.R.E., M.I.E. (India), Professor of Electrical Machine Design. Complete construction was done in the workshop attached to the Electrical Engineering Labomatory of the college of Engineering, Guindy.

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C O N T E N T S.

CHAPTER I.

S C O P E.

The Induction Motor is the most extensively used of all alternating current Motors. In its simplest form it admits of robust mechanical construction, and its ruggedness and ability to stand rough usage make it a most desirable type of industrial motor. Probably at least 80 per cent of the world's A.C. Motors are plain, polyphase Induction Motors. (R.1) In America about one half of the Induction Motors of 1 H.P. and over have cage machines. This shows the importance of cage rotor. It consists essentially of a stationary Member, called the Stator, and a rotating Member called the rotor. The active part of the Stator consists of a core of laminations of sheet steel of about 0.5 M.M. in thickness. These laminations are slotted on their inner periphery, and assembled in a steel yoke.

In these slots is placed a winding of the required number of phases which may be of the concentric type, of the mush type, or of the barrel type with diamond shaped coils. (R.2.)

The rotor may be of the Squirrel cage type, consisting of bars of copper or aluminium placed in the rotor slots, and connected at each end by a solid ring of a copper, aluminium or brass.

If the starting requirements are such as to demand large starting torque with low starting current, then a rotor of the wound type with slip rings will be used and starting resistances will be used. This rotor is usually of the three phase type with either mush or diamond coils. While the Stator may be of single phase or poly phase type, the rotor is always of the Polyphase type (B.1).

The Single Phase Induction Motor is in very wide use in industry, especially in the fractional horse power field and is extensively used for oil burner, fans, blowers, office appliances, and certain types of small tools. It is also widely used for refrigerators, pumps, compressors, washing and ironing machines. (B.4). For use on the farm, it is used in sizes of from 1 to 7.5 h.p; from threshing, feed griding, corn-shelling, grain drying, corn-husking, hay baling, Milking, Water pumping, bottle washing, Milk cooling and also for Silo work. It is a very useful motor in relatively small outputs. For large powers it suffers from disadgantages, which are inherent in its characteristics, and is never used in cases where a poly phase motor can be adopted. Chief among these disadvantages are: (a) out put only about 50 per cent of the three phase Motor, for a given frame size and temperature rise (b) lower Power factor, (c) lower efficiency, and (d) has no inherent starting torque, and, therefore requires a starting winding with a phase splitting device. In spite of these drawbacks, it is admirably adapted for small outputs. Whilst the simplest in construction for all a.c. Machines, its

theory is more complicated than any. Its characteristics are deduced by two very different methods. The methods are known as (1) the cross field theory and (2) the two-revolving field theory (B.3.)

There is an enormous field for fractional h.p. motors, and it maybe desirable to indicate some of the types which are available. They are built for single phase, two and three phase supply circuits. There are three types of Single phase motor available, namely the "general purpose" split phase, the "high torque" split phase, and the capacitor-start motor.

The general purpose Split phase motor is built for 110 to 225 per cent starting torque and 200 to 280 per cent break down torque in relation to full load torque. Its applications include oil burners, fans, blowers, office appliances, and certain types of small tools. It is built in outputs of from 1/20 to 1/3 h.p. for 60, 50 and 25 c/s and 115 to 230 v and speeds of 2850, 1425, 960 r.p.m. are available on 150 c/s.

The Split-phase high torque motor has 200 to 300 per cent. Starting torque with 260 to 350 per cent break-down torque in relation to full load torque. Its application include washing and ironing machines. It is usually built for 1/6, 1/4 and 1/3 h.p. on 60, 50 and 25 c/s for 115 to 220 V. Approximate speeds are 1725 r.p.m. on 60 c/s

1425 on 50 c/s and 1425 on 25 c/s.

Capacitor start Motors are built for 300 to 450 per cent starting torque, with 225 to 300 per cent breakdown torque with low starting current. They are built in sizes varying from 1/6 to 3/4 h.p on 60, 50 and 25 c/s with speeds of 2850, 1425 and 960 r.p.m. on 50 c/s. The size of the Capacitor is 70 to 80 micro.f. for 1/8 H.P. 120 to 150 micro. f. for 1/4 h.p. and 230 to 285 micro. f. for ½ H.P. The starting winding is opened by a centrifugal switch at approximately 70 percent of synchronous speed. The starting torque is of the order of 3 to 3.5. OZ ft/A of starting current at 110 V. These machines have high efficiency and power factor and are ideal for compressors, pumps, stokers, refrigerators, and air-conditioning equipment.) (B.4);

It should be stated that the starting winding, with its capacitance in series, is sometimes left permanently in circuit when running. The motor is then operated as a rather unbalanced two phase motor. When so used, the capacitance used is much smaller than when used for starting purpose only and is of the order of 3 to 15 micro. f. The starting torque is much smaller in this case and of the order of 40 to 50 per cent full load torque. An electrolytic capacitor is used for starting purpose, and a paper capacitor for permanent running (B.2)

Polyphase motors of the Squirrel cage type are built for outputs in the fractional h.p. range of 1/6 to 3/4 h.p. for 2, 4, and 6 poles.

CHAPTER II.

HISTORY OF THE INDUCTION MOTOR.

The discovery by Gambey, the instrument- maker of Paris, that a compass needle, when disturbed and set oscillating, comes to rest more quickly when it is in the vicinity of copper, than when wood is near it, was made in 1824. At that time also Barlow and Marsh, at Woolwich, had observed the effect on a magnetic needle of rotating it near a sphere of iron. Arago published, in 1824, on account of an experiment with a compass needle within rings of different materials. In this experiment he pushed the needle aside to about 45° and counted the number of oscillations made by the needle before the swing decreased to 10°. With a ring of wood the number of oscillations was 145; with a copper ring 66; and with a stout copper ring only 33. In 1825 he suspended a compass needle over a rotating copper disc and found that, by turning the disc slowly, the needle is deviated out of the magnetic meridian. By rotating the disc fast enough he found that continuous rotation of the needle could be produced. The brilliant discovery by Faraday, in 1831, of electromagnetic induction provided the solution to the question of the origin of forces present in the above experiments of Gambey, Barlow and Marsh, and Arago. Faraday showed that the rotation of the Arago disc was due to induced currents, set up in the disc by relative motion of disc and compass needle. From 1831 to 1879 this valuable discovery produced no further results.

In June, 1879, Mr. Walter Baily read a paper, before the Physical Society of London, on " A mode of Producing Arago's Rotations." Baily used a fixed electromagnet with four magnet cores joined to a yoke.

The four magnet cores were about 4 in. long and each was wound with about 150 turns of insulated copper wire of 2.5 m.m. diameter. The coils were connected two and two in series, similar to two independent horse-shoe magnets and were set diagonally across one to another.

The two circuits were connected separately to a revolving commutator, built up of a simple arrangement of springs and contact strips mounted on a piece of wood, with a wire handle by which it was turned. By rotation, the currents from two batteries were caused to be reversed alternately in the two circuits, and this gave rise to the following ghanges in polarity of the four poles.

N O	N N	O N	S N	S O
O ↘ S	S ↓ S	S ↙ O	S ← N	O ↖ N

In this rotating magnetic field a copper disc was suspended. He stated: " The rotationof the disc is due to that of the magnetic field in which it is suspended, and we should expect that, if a similar motion of the field could be produced by any other means the result would be a similar motion of the disc." He also suggested that if a whole circle of poles were arranged under the disc, successively excited in opposite pairs, the series of impulses all tend to make the disc revolve in one direction around

the axis, and added : " In one extreme case, when the number of electromagnets is infinite, we have the case of a uniform rotation of the magnetic field, such as we obtain by rotating permanent magnets." It is clear that Mr.Baily had grasped the fundamental principle of action of the induction motor, and the motor he exhibited before the Physical Society, in 1879, was the first induction Motor, but it needed later important discoveries of methods for producing the revolving field by means of alternating currents to make it the useful machine that is today. The next discovery was made by Marcel Deprez in 1883.

Deprez fed alternating current to a coil, which produced an alternating or oscillating field along the Ox axis. He supplied another coil, whose magnetic axis made an angle of 90° with the oX axis, with alternating current, whose phase difference was 90° in time from the current in the first coil, and showed that a revolving field of constant amplitude could be produced. The frequency of the two currents was the same. He also showed that if the two currents were of equal period, but not of equal amplitude, an elliptically rotating field was produced. The number of turns in each coil was the same.

Professor Ferraris arrived at the same conclusions as Baily and Deprez in 1885, and apparently without knowing of the work of either. His paper on " Electrodynamic Rotations produced by Means of Alternating Currents" was published in 1888. He suggested the method of obtaining currents, differing in phase by nearly 90°, by inserting a resis-

tance in one winding and inductance in the other, thus making the ratio of $\frac{\text{reactance}}{\text{Resistance}}$ small in one winding and large in the other. This method, it may be noted, is largely used for starting up single phase motors.

Then followed the great work of Nikola Tesla between 1887 and 1891. His researches placed the induction motor on a sound foundation. His patents were sold to the Westing House Company of America, whose pioneer efforts in this field must be recognised. In that period, however, the only ac. supply circuits were single phase, and the frequencies were 133 and 125 c/s. These supply circuits were obviously unsuitable for the development of the motor.

In 1891 the Electrotechnical Exhibition at Frankfort was held, and three phase transmission of power was demonstrated. Two turbine-driven three-phase generators were installed at Lauffen, generating 1400 A at 55 V. The frequency was 40 c/s. Three phase transformers were installed, at each end of the line, to raise the voltage to 8000 V at Lauffen, and to reduce it to 65V at Frankfort. The distance of transmission was 110 miles. This bold experiment demonstrated the feasibility of three-phase transmission of emrgy. The load consisted of a 100 hp. three-phase motor and also lamps. Several German firms exhibited different types of three phase induction motors at the exhibtion. One three-phase, 36 h.p motor had the three-phase supply brought into the rotor by three slip rings, the secondary circuit, being the stator winding, consisted of a closed-circuit winding. Several motors, built by Oerlikon Co.,

and designed by the late C.E.L. Brown, were shown. One such motor was a threephase, 20 b.h.p. motor. It had a distributed stator winding and a squirrel cage rotor, and a small airgap. The squirrel-cage rotor was the invention of Mr. Dolivo-Dobrowolky, who co-operated with Mr. Brown in the design of these motors. This motor of Brown's closely resembled in construction the motor of today. From the short account given, it will be realized that much progress was made purely as the result of experiment, and that much theoretical investigation was needed to explain the reactions taking place in the motor and also to show how it could be designed to give the characteristics desired. To that end it was necessary to give a lucid theroy of alternating currents. Thomas H. Blakesley gave a series of ten brilliant papers in the Electrician in 1885. He discussed, for the first time, alternating current phenomena by means of polar diagrams. Then followed the work of the late Professor Gisbert Kapp in his papers contributed to the Institute of Civil and Elecgrical Engineers in 1890.

In 1892, F. Bedell and A.C. Crehore published their book, " Alternating currents, " in which polar diagrams were used and applied to the theory of the transformer and the locus of the primary emf of the current transformer was shown to be a circle. In 1894, Kapp gave a very lucid demonstration of elementary account of the phenomena in induction motors. The polar diagram was developed and included the primary resistance and a leakage. His diagram was given for each point of the load, and gave no general solution

showing how the different characteristics varied with the load.

In 1895, Blondel gave his papers, on " Some General Properties of Revolving Magnetic Fields" In these papers, published in Eclairage Electrique, he unfolded the theory of the composition of magnetic fluxes, including leakage fluxes. In 1895, also, the late Mr. B.A. Behrend proved that the locus of the primary current of the alternating current transformer is a circle in the Polar diagrams, provided the primary resultant magnetic field is constant. The circle locus of the induction motor was also shown by A. Heyland in 1894. There has been much controversy about priority in this discovery. Part of the credit must be given to Dr. Bedell, who stated: " In any circuit or apparatus with constant reactance and variable power consumption, the current will have a circle locus if the supply voltage is constant". This was first shown by Bedell and Crehore in 1892.

Bedell also states: " That the induction motor nearly fulfils those conditions and that its current locus is practically the arc of a circle, was first shown by Heyland in 1894." It is also stated that Kapp and Behn-Esche berg first pointed out the identity of the theory of the a.c transformer and the induction motor in 1892 and 1894.

The Circle diagram in use today is undoubtedly due to Behrend, and one is impressed with the beautiful simplicity of the diagram, and the ease with which the characteristics of the motor are determined from it, commends it to the

designer.

Although the relation of dimensions to characteristics was known about 1900 or so, it remained to determine the effect of harmonics, due to the distribution of the winding, and also due to slots, on the performance of the motor. Analysis of the m.m.f diagrams showed that several rotating fields were produced, the fundamental and various harmonics. These fields rotate at different speeds with respect to the rotor, and some rotate in the same direction, some in the opposite direction, and produce both driving and retarding torques. Such harmonics may, and do, produce noise and vibration and may, by producing saddle backs in the torque curve, prevent the machine from accelerating to full speed. Especially is this true in pole-changing motors. This question of noise has been investigated by H. Fritze and Kron and Chapman. A large number of papers has been written on this very important subject .

The question of improved efficiency has resulted in improvements in the manufacture of steel laminations, by the introduction of silicon in various per centages. Brands known as Stalloy and Super-stalloy have been introduced, and are used in those cases where it is necessary to keep down in the iron losses, in totally enclosed machines, and especially in machines for 400 c/s, such as are used in connection with the automatic pilot for planes.

Then the question of eddy-current losses in conductors has been thoroughly investigated by A.B. Field and others.

The trend is towards greater and greater output from a given mass of materials, and this can only be effected by scientific design and proper proportioning of the machine. The introduction of Silicone Varnishes for insulation has removed the conservative temperature rises formerly allowed and resulted in smaller machines for a given output and speed.

Ventilation is another problem, which has received much attention and is one of the most important factors in increasing output from a given frame. Much more research is still needed on this important question.

The question of Speed control in induction motors has received much attention. The induction motor is essentially a constant speed machine, like the d.c shunt motor and this is, in some case, rather a serious draw back. There are many industrial applications where speed control is necessary and the induction motor is the ideal motor for many such application being simple, rugged and reliable. but various methods must be adopted to secure efficient speed control which spoil its simplicity. The methods adopted are (1) Pole changing (2) cascade connection; (3) cascading with commutatoε motor; (4) change of frequency supply.

The induction motor is now the most widely used of all machines. It is doubtful whether the large power system, now in such extensive use, would have been developed if this motor had not been developed. What it means to the economy of the world is appreciated by few people outside the engineering world. In the fractional horse-power field, its develop-

ment is phenomenal and it is applied in every form of industrial and domestic work. In this field it takes the form of Single Phase and Three-Phase type.

CHAPTER III

The Rotating Magnetic Field Theory.

3.1. General Discussion.

In its general form, the a.c. motor comprises one or more stationary circuits and one or more rotating electric circuits inductively related to the stationary circuits. Energy is transferred from the stationary to the rotating circuits through the medium of a common magnetic field. In this discussion the magnetic field is in general considered the resultant of two fields rotating in opposite directions. These two components are not necessarily equal, in fact one of them may vanish and the s resultant is then a uniformly rotating field, as for example im a polyphase machine. Only motors in which the magnetic reluctance is uniform in all directions will be considered, that is, motors with projecting poles are not included.

There are then two main groups of motors to be discussed, the induction type, in which the rotor circuits are short circuited upon themselves, and the commutator type, in which the rotor circuits are either short circuited or connected to an external circuit through the commutator. A number of essential features are common to both types. Thus at any speed the m.mfs of the rotor rotate with velocities which, combined with the velocity of mechanical rotations, are always equal to the velocities of stator fields. The common magnetic field is produced by an m.m.f which is the vector sum of the stator and rotor mmfs.

When running at any slip, s, there are in general two voltages induced in the rotor circuits of frequencies Sf and (2-S) f, according to whether it is induced by the field rotating in the same or in opposite direction to the rotation of the rotor, where f is the stator frequency. In magnitude the induced voltages are of course proportional to their respective frequencies. As this view point obviously eliminates the necessity of considering the rotational and transformer voltages. Separately, the solution of many problems are greatly simplified thereby. The torque developed by either field is also readily determined, it being in all cases the product of the field, m.m. f of the rotor circuits revolving in the same direction as the field and the sine of the angle between them.

As already stated, the voltages induced in the rotor circuits are of frequencies Sf and (2-S) f. In the induction type the rotor currents resulting from the voltages are likewise of frequencies Sf and (2-S) f. However in the commutator type the resulting rotor currents are at all speeds converted by the commutator into line frequency. In their magnetic reactions the rotor currents become fixed in space by the commutator and the rotor circuits can therefore be considered as remaining stationary in every respect, except as regards the magnitudes of the induced voltages, which are determined by the frequencies of slip as pointed out.

The rotor currents becoming fixed in space by the commutator is the cause of some essential differences in the operation of the induction and commutator types. Thus rotor

Thus in Induction machine, the reactance of the rotor circuit changes with the slip, while in the commutator type it is the same at all speeds. Also, by shifting the brushes on the commutator, the relative position of the stator and rotor m.m.f's is changed so that the voltages that are induced in the stator circuits by the rotor currents may be advanced or retarded in time. This becomes of importance when an external emf is impressed on the rotor as it introduces the possibility of Power factor correction.

The subject of Power factor correction has lately been the object of considerable discussion and the readiness with which the rotating field theory lends itself to the analysis of just such problems; involving a displacement angle between the stator and rotor currents is another marked advantage of this method. Application of the method to the analysis of some proposed schemes for Power factor correction will be taken up in mathematical section.

However, before taking up the mathematical discussion, it will be shown by way of illustration how readily the method can be applied to the much discussed problem of calculating the performance of a single phase Induction motor.

Consider a single phase motor and 2 two phase motor of the same constants per circuit as the single phase machine. Then if E be the impressed voltage and I_p the current per phase of the two phase motor at slip s, the apparent impedance per phase is $Z_a = \frac{E}{I_p}$. Similarly if I_b is the current per phase when running backwards at the same

speed, the apparent impedance, at slip (2-S) is $Z_b = \frac{E}{I_b}$

It is then shown in the appendix that the apparent impedance of the single phase motor at slip s is

$$Z_s = \frac{Z_a + Z_b}{2}$$

and the current of the Single phase motor is $I_s = \frac{E}{Z_s}$. Let T_a = Torque of 2 phase motor at slip s and T_b = Torque at slip (2-S) The torque developed in the single phase motor at slip s by the field revolving in the same direction as the rotor then is

$$T_1 = \frac{e_1}{E}^2 \times T_a$$

where

$$e_1 = \frac{Z_a I_s}{2}$$

and the torque developed by the oppositdly rotating field.

$$T_2 = \frac{e_1}{E}^2 \times T_b$$

where

$$e_2 = \frac{Z_b I_s}{2}.$$

The resultant torque of the single phase motor at slips.

$T_s = T_1 - T_2.$ Here from the remaining quantities, power, efficieney and power factor can be determined directly. Thus, by extremely simple calcultions the performance of a single phase motor is derived from the performance of a two phase motor is derived from the performance of a two phase motor of same constants per circuit, Which one of the numerous methods that have been devised for calculating the performance of a 2 phase motor to use is, of course, a matter of choice.

When it is desired to calculate the performance from the running and locked test readings, the locked single phase readings can be used directly in calculating the two phase performance. The single phase no load reading with the rotor short circuited, that is, running light, cannot be so used, because the exciting admittance per pahse of the two phase motor is then a little more than one half the single phase admittance, or, what amounts to the seme thing, the two phase no load impedance per phase is almost twice the single phase no load impedance. The amount by which the two phase imoddance falls short of being exactly twice the single phase no load impedance is obviously the apparent impedance per phase when running backwards at full speed. Since this latter impedance is almost independent of the exciting current it can readily be determined, to a high degree of accuracy. On the basis of the modified single phase no load reading the diagram of the two phase motor dan be constructed and the single phase performance calculated therefrom.

3.2 <u>Mathematical Expression for two oppositely Rotating fields</u>

A current with maximum value I and varying periodically according to the cosine law is usually represented by the expression I Cost wt where w = 2 II f, f being the frequency Using the exponential form of the cosine a sinusoidal current may be represented by a pair of rotating vectors, that is I Cos wt may be written

$$\frac{I}{2} E^{jwt} + \frac{I}{2} E^{-jwt}$$

Many cumbersome trigonometric expression and transformations are frequently avoided by the use of this notation. That is the expression $\frac{I}{2} E^{Jwt}$ represents a vector revolving in counterclockwise direction with angular velocity w is readily seen by writing $\frac{I}{2} E^{Jwt} = \frac{I}{2}$ (Cos wt + J Sinwt) and assigning to t a series of increasing positive values. The Expression

$$\frac{I}{2} E^{-Jwt} = \frac{I}{2} \left(\text{Cost } wt - J\text{Sin } wt \right)$$

is likewise seem to represent a vector of same length and revolving in clockwise direction. In the figure let a current I Cost wt be flowing in a coil NN of n turns. The instantaneous value of the m.m.f of the coil then is nI Cos wt or in the above notiation

$$\frac{n\,I}{2} E^{Jwt} + \frac{n\,I}{2} E^{-Jwt}$$

As the m.m.f is a directed quantity in space the first term of this expression represents a m.m.f of constant intensity $\frac{n\,I}{2}$ rotating in positive direction, and the second term represents a m.m. f of same intensity rotating in opposite direction. Let the coil N-N be turned through an angle of Ø radians in positive direction and its m.m.f becomes

$$\frac{n\,I}{2}\,(E^{Jwt} + E^{-Jwt})\quad E^{J\phi}$$

$$\frac{n\,I}{2}\,E^{J\,(wt+\phi)} + \frac{n.\,I}{2}\,E^{-J(wt-\phi)}$$

that is, the m.m.f revolving in positive direction has been advanced Ø radians and the m.m.f revolving in negative direction has been retarded Ø radians.

The most striking feature of the poly phase system, that it can produce a rotating magnetic field of constant intensity, is most readily deduced by the use of thisnotation.

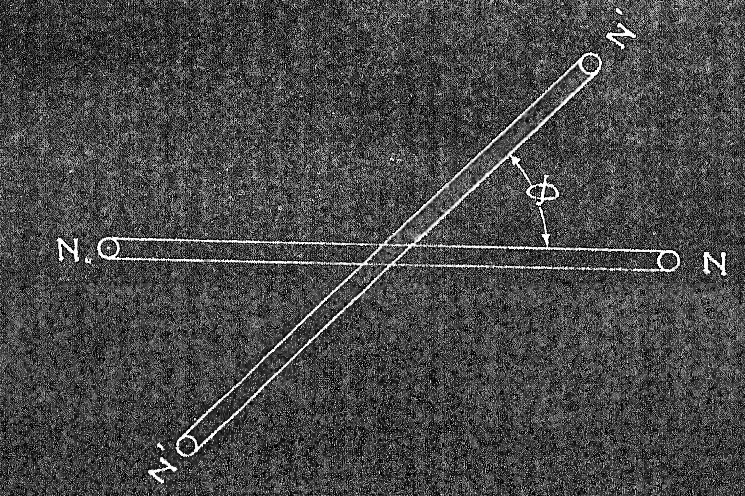


FIG. Nº 1.

To illustrate assume m coils, symmetrically based on a cylindrical core, the space angle being $\frac{2\,\Pi}{m}$ radians. The coils are excited by currents I, differing in phase by $\frac{2\,\Pi}{m}$ radians. The m.m.f of the k^{th} coil is then,

$$\frac{n.I.}{2}\left[E^{J(wt+2\pi k/m)} + E^{-J(wt+2\pi k/m)}\right]E^{J\,2\pi k/m}$$

and the total m.m.f of all the coil is

$$\sum_{0}^{n-1} k\ \frac{M.I}{2}\left[E^{J(wt+2\pi k/m)} + E^{-J(wt+2\pi k/m)}\right]$$

$$= \sum_{0}^{m-1} k\ \frac{n.I}{2}E^{J(wt+4\pi k/m)} + \sum_{0}^{m-1} k\ \frac{nI}{2}E^{-Jwt}\ E^{ok}$$

$$= \frac{mnI}{2}E^{-Jwt} \quad \text{since} \quad \sum_{0}^{m-1} k.\ \frac{nI}{2}\ E^{J(wt+4\pi k/m)} = 0$$

That is the resultant m.m. f of the constant intensity mnI/2. and rotating in negative direction. If the sign of either time angbk or space angle be changed the resultant m.m.f is seen to rotate in opposite direction.

In the previous figure let Zm be the mutual impedance between coils NN and N'N' when in co-axial position. The Voltage induced in the coil N'N' by the current flowing in NN then is Zm I. When N'N' is turned Ø radians in positive direction the voltage induced by the positively rotating field of NN is retarded Ø/ radians, and becomes $\frac{Zm\ I}{2}E^{-J\emptyset}$ and the voltage induced by the negatively rotating field is advanced Ø radians and becomes $\frac{Zm\ I}{2}E^{J\emptyset}$. The voltage induced by both fields is

$$\frac{Zm\ I}{2} \varepsilon^{-J\emptyset} + \frac{Zm\ I}{2} \varepsilon^{J\emptyset} = Zm\ I \cos\emptyset$$

Single Phase Induction Motor.

The rotor circuits of the single phase induction motor being short circuited upon themselves in all directions constitute a polyphase system and for convenince will be considered two phase with constants determined accordingly. Assume the rotor to be revolving at slip s and consider its direction of rotation positive. Using the customary notations let $Zm = r\,m + J\,xm$ = mutual inductive impedance between stator and rotor circuits.

$Z_o = r_o + J\,x_o$ = Self inductive impedance of primary.

$Z_1 = r_1 + J\,s\,x_1$ = Self inductive impedance to rotor current of frequency $S\,f\,x1$

x_1 = Secondary reactance at standstill in terms of primary.

$Z_2 = r_1 + J\,(2-S)\,x_1$ = Self inductive impedance to rotor current frequency $(2-S)f$.

E = Volts impressed on primary.

I_s = Primary current.

I_1 = positively rotating component of rotor current.

I_2 = Negatively rotating component of rotor current.

The voltages induced in the primary then are: by the primary current.

$I_S + = (Z_m + Z_o)\,I_s$ by the positively rotating rotor current $Z_m\,I_1$ by the negatively rotating rotor current $Z_m I_2$. the currents I_1 and I_2 being two phase have full inductive effect.

The sum of these voltages equals the impressed Volts, hence,

$$(Z_m + Z_o) I_s + Z_m I_1 + Z_m I_2 = E$$

The voltages induced in the rotor are:

by the positively rotating component of primary current

$$\frac{s Z_m I_s}{2}$$

by the positively rotating component of rotor current

$$(s Z_m + Z_1) I_1$$

by the negatively rotating component of primary current

$$\frac{(2-S) (Z_m) I_s}{2}$$

by the negatively rotating component of rotor current

$$\left[(2-S) Z_m + Z_2)\right] I_2$$

The sums of voltages of frequencies Sf and (2-S) f are separately equal to zero, hence

$$\frac{S Z_m I_s}{2} + (s Z_m + Z_1) I_1 = 0 \quad \text{or}$$

$$S Z_m I_s + 2 (S Z_m + Z_1) I_1 = 0 \quad \text{and}$$

$$\frac{(2-S) Z_m I_s}{2} + \left[(2-S) Z_m + Z_2)\right] I_2 = 0 \quad \text{or}$$

$$(2-S) Z_m I_s + \left[(2-S) Z_m + Z_2)\right] I_2 = 0$$

The e.m.f. equation of primary and secondary are then

$$\left.\begin{array}{l} (Z_m + Z_o) I_s + Z_m I_1 + Z_m I_2 = E \\ S Z_m I_s + 2 (S Z_m + Z_1) I_1 = 0 \\ (2-S) Z_m I_s + 2 \left[(2-S) Z_m + Z_2)\right] I_2 = 0 \end{array}\right\} \quad (1)$$

solving these equations.

$$I_s =$$

$$\frac{E2\left[S(2-s)Z_m^2+(2-S)Z_mZ_1+sZ_mZ_1+Z_1Z_2\right]}{2Z_o\left[S(2-S)Z_m^2+(2-S)Z_mZ_1+SZ_mZ_2+Z_1Z_2)\right]+Z_m\left[(2-S)Z_mZ_1+SZ_mZ_2+2Z_1Z_2)\right]} \quad \ldots(2)$$

$$I_1 =$$

$$\frac{-EsZ_m\left[(2-S)Z_m+Z_2)\right]}{2Z_o\left[S(2-S)Z_m^2+(2-S)Z_mZ_1+SZ_mZ_2+Z_1Z_2)\right]+Z_m\left[(2-S)Z_mZ_1+SZ_mZ_2+2Z_1Z_2)\right]} \quad \ldots(3)$$

$$I_2 =$$

$$\frac{-E(2-S)Z_m(SZ_m+Z_1)}{2Z_o\left[S(2-S)Z_m^2+(2-S)Z_mZ_1+SZ_mZ_2+Z_1Z_2)\right]+Z_m\left[(2-S)Z_mZ_1+sZ_mZ_2+2Z_1Z_2\right]} \quad \ldots(4)$$

The equations of a 2 phase motor of same constants perphase are easily found to be

$$(Z_m+Z_o)I_o+Z_mI_1' = E$$

$$SZ_mI_o+(SZ_m+Z_1)I_1' = 0$$

where I_o = primary current per phase

and I_1' = Secondary current.

Here from

$$I_o = \frac{E(SZ_m+Z_1)}{SZ_mZ_o+Z_mZ_1+Z_oZ_1} \quad \ldots(5)$$

$$I_1' = \frac{-E\ S\ Z_m}{S\ Z_m Z_o + Z_m Z_1 + Z_o Z_1} \qquad ..(6)$$

The apparent impedance per phase of the two phase motor at slip s then is

$$Z_a = \frac{E}{I_o} = \frac{S\ Z_m Z_o + Z_m Z_1 + Z_o Z_1}{S\ Z_m + Z_1}$$

The apparent impedance at same speed rotating backwards is

$$Z_b = \frac{E}{I_o} = \frac{(2-S)\ Z_m Z_o + Z_m Z_2 + Z_o Z_2}{(2-S)\ Z_m + Z_2}$$

Adding the apparent impedances of both rotations

$$Z_a + Z_b = \frac{S\ Z_m Z_o + Z_m Z_1 + Z_o Z_1}{S\ Z_m + Z_1} + \frac{(2-S)\ Z_m Z_o + Z_m Z_2 + Z_o Z_2}{(2-s)\ Z_m + Z_2}$$

$$= \frac{2\ Z_o \left[S\ (2-S)\ Z_m^2 + (2-S)\ Z_m Z_1 + S\ Z_m Z_2 + Z_1 Z_2\right]}{S\ (2-S)\ Z_m^2} +$$

$$\frac{Z_m \left[(2-S)\ Z_m\ Z_1 + S\ Z_m Z_2 + 2\ Z_1 Z_2\right]}{+ (2-S)\ Z_m\ Z_1 + S\ Z_m Z_2 + Z_1 Z_2}$$

Comparing with (2) it will be seen that the apparent impedeance of a single phase motor at slip S is one half the sum of the apparent impedance of a two phase motor of same constants at slips s and (2-s). Further more, if the impedance drop $\frac{Z_a\ I_s}{2}$ be substituted for E in (6) the result is identical with

(3) showing that the low frequency component of secondary current is equal to the two phase secondary current at this reduced voltage. Similarly the high frequency component of secondary current is seen to be equal to the two phase secondary current at slip (2-s) and the voltage reduced to $\frac{Z_b I_s}{2}$. The torque components of single phase motor is positive and negative directions are likewise seen to be equal to the torques of the two phase machine at correspo nding slips and reduced voltages.

It follows therefore that in calcualting the performance by the geographical method there is no need of a special diagram for the single phase motor, the same diagram being applicable to both polyphase and single phase motors.

***

CHAPTER IV.

The Cross Field Theory.

Alternating current machines may be analyzed according to either the revolving field or the cross field theory. Each of these theories has its own individual merits depending more or less on the type of machines under consideration. Some phenomena are more easily understood by a study following the revolving field theory, and other phenomena are made more clear by the use of the cross field theory (J.1).

As for routine calculation of performance characteristics of alternating current machines, many graphical and analytical methods have been developed. These different methods also have their own individual merits and the choice of a method of calculation should depend partly on what is most desired; e.g speed, accuracy or aid to visualization.

Therefore, we may say that, in general, neither the revolving field nor the cross field method of analysis and no one method of calculation, graphical or analytical should be used exclusively. Although this chapter deals only with a general analytical method of studying some types of a.c. machines using the cross field theory. it is not by any means intended as a plea for the exclusive use of this general method for it is recognised that whatever usefulness it may have will be found in rather limited fields. (J.5)

In the following, the attempt is made to show how a general method of analysis, following the cross field theory may be applied to alternating current motors to obtain simple and accurate, purely numerical methods for routine calculations of performance characteristics. The fundamental principles of the analysis of motors by the cross field method are, of course, very well known. The general method given below is fundamentally the same as that outlined by Steinmetz in his "Theory and Calculation of Electrical Apparatus," but differs from it in the treatment of the leakage reactance, and in the arrangement of the results.

Briefly stated, the general method as applied to a motor is as follows: Kirchoff's voltage equations are set up for different circuits of the motor and are solved to obtain equations for the currents in each of the circuits in terms of the applied volatage, the design constants of the motor and the speed. From these equations, other equations for the fluxes linking or cut by the rotor conductors are obtained. The torque corresponding to any one of the rotor circuits is obtained by multiplying the inphase components of the currents and fluxes cut by the conductors of that circuit. Adding the components of the torque corresponding to each of the rotor circuits, we obtain an expression for the total torque developed, which multiplied by the field, gives the power generated. Subtracting from this the friction losses gives the Power output. The power input is of course, given by the product of the applied voltage and the in phase component of the line current. (J.2).

In this chapter, the anlge of the hysteretic lag between flux and m.m.f is neglected in the analytical solutions, and correction for core loss is made in the Numerical calculations by treating the core loss the same as if it were a friction loss. If it should be desirable the angle of hysteric lag can be taken into accout by using the complex quantity.

$Z_m = R_m + J X_m$ for the magnetizing impedance in phace of the Pure reactance JX_m which is used in the equations in the following part of this chapter. This would complicate matters slightly by adding to the lengths of the equations and would yield but a very slight increase in accuracy. In almost all cases, the slight increase in accuracy would not justify the extra labour and chances for numerical error.

Sine wave distribution of m.m.f and uniform air gap permeance are assumed in all cases.

In details and application of the method can best be shown by means of examples as follows.

The Single Phase Induction Motor.

According to the cross field theory, the components of the main flux of the motor and rotor currents are considered separately in two axes at right angles to each other. The axis of the stator winding may be called the transformer axis, and the axis at right angles to it the field axis. A squirrel cage is considered as equivalent to a commutated

winding with brushes bearing on the commutator short circuited on themselves in the transformer and field axes. The motor can then be represented diagramatically as in fig. 1.

The following symbols will apply

E = applied voltage.

I_1 = Line current

I_{1a} = Power component of line current.

I_{1b} = Reactive component of line current.

I_{2t} = rotor current in the transformer axis.

I_{2f} = rotor current in the field axis.

r_1 = resistance of the stator winding

r_2 = resistance of each of rotor circuits

X_m = mutual inductive reactance of the stator and rotor windings.

x_1 = leakage reactance of the stator winding

x_2 = leakage reactance of each of the rotor circuits.

N = effective number of turns in each of the circuits

f = frequency of applied voltage

S = speed as a farction of synchronism.

The symbols for voltage and current all represent r.m.s. values of time vector quantities. The positive senses of the currents are indicted by the arrows in Fig.2.

The motor flux is resolved into the following four components: (1) the transformer flux ϕ_{mt} which is the flux that is mutual to the stator winding and the rotor circuit, in the transformer axis; (2) the field flux ϕ_f which is the flux produced by the m.m.f of the current I_{2f} in the field axis of the rotor; (3) the leakage flux ϕ_1 of the primary and (4) the leakage flux ϕ_{2t} of the rotor circuit in the transformer axis.

In terms of the current and the motor reactances the equations for the above flux components are:

$$\phi_{mt} = \frac{X_m \quad (I_1 - I_2 t)}{2\pi f N} \tag{1}$$

$$\phi_f = \frac{X_m + X_2 \quad I_2 f}{2\pi f N} \tag{2}$$

$$\phi_1 = \frac{x_1 \ I_1}{2\pi f N} \tag{3}$$

$$\phi_{2t} = \frac{x_2 \ I_2 t}{2\pi f N} \tag{4}$$

Voltage applied to the stator terminals must overcome the resistance drop and the mutual and leakage reactance drops due to alternation of ϕmt and $\phi 1$. The equation is

$$E = r_1 I_1 + j x_1 I_1 + j X_m (I_1 - I_2 t)$$

In the transformer axis of the rotor, the sum of the voltages induced by transformer action of ϕmt and $\phi 2t$ and by rototation through the flux ϕf plus the resistance

drop r2I2t must equal zdro. The equation is, for counter clockwise rotation of the r otors (J.3)

$$0 = -j X_m (I_1 - I_{2}t) - S (X_m + x_2) I_2 f$$
$$+ r_2 I_2 t' + j x_2 I_2 t \quad \ldots (6)$$

Similarly for the field axis of the rotor,

$$0 = j (X_m + x_2) I_2 f - S X_m (I_1 - I_2 t) +$$
$$S x_2 I_2 t + r_2 I_2 f \quad \ldots (7)$$

Solving these three voltage equations for

I1, I2t and I 2f we get,

$$I_1 = E \frac{E \quad r_2^2 + (1 - S^2) (X_m + x_2)^2 - j2r_2(X_m + x_2)}{U_1' + J W_1'}$$

$$I_{2t} = \frac{E \quad X_m (1 - S^2) (X_m + x_2) - J r_2}{U_1' + J W_1'}$$

$$\ldots (9)$$

$$I_{2f} = - \frac{SE X_m r_2}{U_1' + J W_1'}$$

where

$$U_1' = - r_1 r_2^2 + 2 r_2 x_1 (X_m + x_2) +$$
$$r_2 X_m (X_m + 2 x_2) + (1-S^2) r_1(X_m + x_2)^2$$
$$\ldots (11)$$

$$W_1' = - r_2^2 x_1 - 2 r_1 r_2 (X_m + x_2) - r_2^2 X_m$$
$$+ (1-S^2) x_1 (X_m + x_2)^2 +$$
$$x_2 X_m (X_m + x_2) \quad \ldots (12)$$

Substituting the above values of I1, I2t and I2f in equations (1), (2) and (4) we get,

$$\phi_{mt} + \phi_{2t} = \frac{- E X_m \left[r_2^2 + J r_2 (X_m + x_2) \right]}{2 \pi f N \; (U_1' + J W_1')} \quad .. (13)$$

and

$$\phi_f = \frac{- S E X_m (X_m + x_2) \; r_2}{2 \pi f N \; (U_1' + J W_1')} \quad ..(14)$$

The torque developed by the motor consists of two components, one component T1 due to the interaction of the current I2t and the flux ϕf and the other component T2 due to the interaction of I2f and the flux)
$\phi_t = \phi_{mt} - \phi_{2t}$. These torque components in synchronous watts are equal to the products of the in phase components of the currents and the fluxes with which they interact multiplied by $2\pi f$ N: that is from equation (9) and (14)

$$T_1 = \frac{E^2 X_m^2 \; (X_m + x_2)^2 \, r_2 \, S \, (1 - S^2)}{U_1'^2 + W_1'^2} \quad .. (15)$$

and from equations (10) and (13)

$$T_2 = \frac{- E^2 \, X_m^2 \; r_2^3 \, S}{U_1'^2 + W_1'^2}$$

and the total torque developed by the motor is

$$T = T_1 + T_2 = \frac{E^2 X_m^2 \, r_2 S \; \left[(1 - S^2) \; (X_m + x_2)^2 - r_2^2 \right]}{U_1'^2 + W_1'^2}$$

This equation shows that the torque developed by the motor is zero when

$$1 - S^2 = \frac{r_2^2}{(X_m + x_2)^2}$$

or ideal no load speed is

$$S_o = \sqrt{1 - \frac{r_2^2}{(X_m + x_2)^2}} \qquad ..(18)$$

substituting the above value for the no load speed in equation (7) we obtain for the no load current,

$$I_o = \frac{2E\,(X_m + x_2)}{2 r_1 (X_m + x_2) + \dfrac{r_2 X_m^2}{X_m + x_2} + J X_m \left[X_m + 2(x_1 + x_2)\right]}$$

For all usual relative values of the design constants this is very closely

$$I_o = -J \frac{2E}{X_m} \left[\frac{X_m + x_2}{X_m + 2(x_1 + x_2)}\right]$$

$$= -J \frac{2E}{X_m} \left[1 - \frac{2x_1 + x_2}{X_m + 2(x_1 + x_2)}\right] \qquad ..(19)$$

It is obvious that by means of this equation the value of the magnetising reactance Xm can be calculated with any desired degree of accuracy from the values of the short circuited and no load currents. (J.4)

The Performance characteristics of a motor of given design constants can be calculated completely by means of equations (8) and (17). The solution of these equations can be reduced to a simple matter of arithmetic by means

Core loss and friction = 600

$$F_1 = \frac{-E\ r_2^{\ 2}}{(X_m + x_2)^2} = -0.0835$$

$$F_2 = E = 220$$

$$F_3 = \frac{-2E\ r_2}{X_m + x_2} = -8.57$$

$$F_4 = r_2 \frac{x_2 X_m - r_1 r_2}{(X_m + x_2)^2} + \frac{X_m + 2x_1}{X_m + x_2}$$

$$= 0.315$$

$$F_5 = r_1 = 0.12$$

$$F_6 = -r_2 \frac{r_2 (X_m + x_1)}{(X_m + x_2)^2} + \frac{2 r_1}{X_m + x_2}$$

$$= 0.0105$$

$$F_7 = x_1 + x_2 \frac{X_m}{X_m + x_2} = 0.789$$

$$F_8 = -E^2 r_2^{\ 3} \frac{X_m^{\ 2}}{(X_m + x_2)^4} = -5.23$$

$$F_9 = E^2 r_2 \frac{X_m^{\ 2}}{(X_m + x_2)^2} = 13{,}800$$

1.	S	1.00	0.98	0.95	0.90	0.80
2	$1-S^2$	0	0.0396	0.0975	0.19	0.36
3	$S(1-S^2)$	0	0.0388	0.0927	0.171	0.288
4	F1	-0.0835	-0.08	-0.08	-0.08	-0.08
5	$(1-S^2)$ F2	0	8.7	21.4	41.7	79.2
6.	M1=(4)+(5)	-0.0835	8.62	21.3	41.6	79.1

7.	N1 = F3	-8.57	-8.57	-8.57	-8.57	-8.57
8.	F4	0.315	0.315	0.315	0.315	0.315
9.	$(1-S^2)$ F5	0	0.005	0.012	0.023	0.043
10.	U1=(8)+(9)	0.315	0.320	0.327	0.338	0.358
11.	F6	-0.0105	-0.010	-0.010	-0.010	- 0.010
12	$(1-S^2)$F7	0	0.031	0.077	0.150	0.284
13.	W1= (11)+(12)	-0.0105	0.021	0.067	0.140	0.274
14	M1U1	-0.0263	2.76	6.98	14.05	28.3
15.	N1W1	0.09	-0.18	-0.57	-1.20	-2.35
16.	M1U1+N1W1	0.0637	2.58	6.41	12.85	25.95
17	N1U1	-2.70	-2.74	-2.80	-2.90	-3.06
18.	-M1W1	-0.0009	-0.181	-1.43	-5.82	-21.66
19.	N1U1-M1W1	-2.70	-2.92	-4.23	-8.72	-24.72
20	$U1^2$	0.0992	0.1025	0.107	0.1143	0.1282
21.	$W1^2$	0.0001	0.0004	0.0045	0.0196	0.0751
22.	$U1^2 + W1^2$	0.0993	0.1029	0.115	0.1339	0.2033
23.	$I_{1a} = \frac{13}{19}$	0.642	25.1	57.5	95.8	127.5
24.	$I_{1b} = \frac{16}{19}$	-27.2	-28.4	-38.0	65.2	121.5
25.	$I_1 = I_{1a}^2 + E_{1b}^2$	27.3	38.0	69.4	116	1.76
26.	Power factor $= \frac{20}{22}$	--	0.66	0.83	0.825	0.725
27.	Power Input = E ± 20	--	5510	12660	2110	28000
28.	S F8	-5.123	-5.1	-5	-5	-4
29.	$S(1-S^2)$ F9	0	535	1280	2360	3970
30.	28 + 29	-5.23	530	1275	2355	3966
31.	T = 30/19	-52.8	5150	11450	17600	19450

32.	Core loss & friction	600	600	600	600	600
33.	Net Torque 31 - 32	-653	4550	10850	17000	18850
34	Power Output 1 = S (33)	-653	4460	10300	15300	14700
35.	Efficiency $\frac{34}{27}$	-	0.808	0-812	0.725	0.532.

It will be noted that all the steps are indicated in the above. It is obvious that if a printed calcubtion form arranged as the above is used, the calculation of the complete performance characteristics of the Single Phase Induction Motor is made comparable to a Simple problem of book keeping, and can be done by a person without any technical training. (J. 5)

The performance corves plotted from the above calculated values for current, torque, power factor, and efficiency are shown in Fig. 3.

CHAPTER V

Single Phase Motor - Theory - a correlation of the Cross field and Revolving Field Concepts.

Introduction.

The revolving field theory of Single Phase Induction motor operation resolves the total flux crossing the air gap into two components rotating in opposite directions but constant in magnitude. The cross field theory resolves the total flux into two components of varying magnitude but constant direction with reference to the stator. (J.1).

The cross field and revolving field theories have been developed by H.R. West and Wayne J. Morrill respectively, in a complete and analytical manner. From the standpoints of the concepts involved and their final utility, each of these presents certain difficulties. The objective of this paper is to utilize and co-ordinate both fundamental methods of analysis. A new treatment is used, however which does not follow the detailed methods or type of development previously used for either theory.

While it is necessary to treat certain factors in an idealized manner, the effort is made to consider a single phase Induction Motor as it actually is, rather than to say arbitrarily that it is equivalent to some fictitious circuit or other mechanical arrangement as is done with some other theories.

The discussion herein will start with the general expression for voltage induced in a coil rotating in a single phase field periodically reversing but not rotating.

A two pole motor will be considered throughout for simplicity.

Rotor voltage.

In a single phase Induction motor there is in practice generally a rough approximation to sinusoidal distribution of flux around the air gap at any instant. In figure 1 it is assumed that the flux density is uniform along the X-X axis. The coil has no magnetic core and is simply turnig in the field shown. Along the circle traced by the end of the coil the flux distribution will then be sinusoidal at any instant.

The flux through the coil will be

$$\phi_c = \phi_M \ \mathrm{Sin}\,\omega t \ \mathrm{Cos}\ \theta \qquad (1)$$

$$= \phi_M \ \mathrm{Sin}\,\omega t \ \mathrm{Cos}\ (\ pwt + \alpha) \qquad (2)$$

$$= \phi_M \ \mathrm{Sin}\,\omega t \ (\ \mathrm{Cost}\ pwt \ \ \mathrm{Cos}\,\alpha - \mathrm{Sin}\ pwt\ \mathrm{Sin}\,\alpha\ ) \qquad ..(3)$$

$$= \frac{\phi_M}{2} \ \cos\alpha \left[\mathrm{Sin}\ (1+p)\ wt + \mathrm{Sin}\ (1-p)wt \right] +$$

$$\mathrm{Sin}\,\alpha \ \ \mathrm{Cos}\ (1+p)wt - \mathrm{Cos}\ (1-p)\ wt \qquad (4)$$

$$= \frac{\phi_M}{2} \ \mathrm{Sin}\left[\alpha + (1+p)wt \right] - \mathrm{Sin}\left[\alpha - (1-p)wt- \right] \qquad (5)$$

$$\frac{\phi_M}{2} \ \mathrm{Sin}\left[(\ 2-S)\ wt + \alpha \right] + \mathrm{Sin}(Swt - \alpha) \qquad ..\ (6)$$

Voltage in coil $= \dfrac{d\phi_c}{dt}$ (time is a constant which is here neglected.)

$$= \frac{\phi_M}{2} \ (2-S)w\ \mathrm{Cos}\left[(2-S)wt + \alpha \right] +$$

$$Sw\ \mathrm{Cos}\ (\ Swt - \alpha\ ) \qquad ..(7)$$

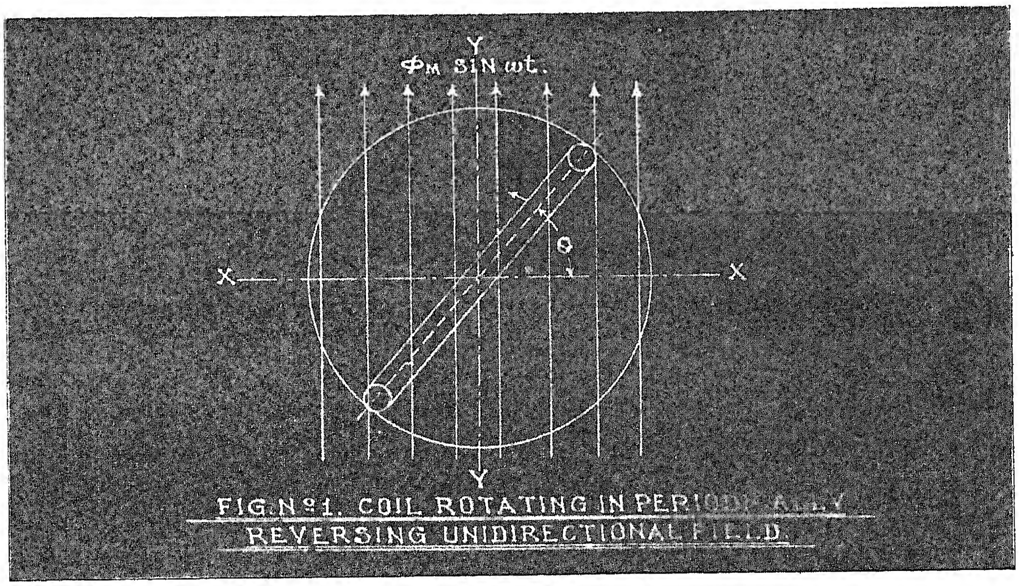


FIG. Nº1. COIL ROTATING IN PERIODICALLY REVERSING UNIDIRECTIONAL FIELD.

It is seen that this expression for the voltage is precisely what we would have if the Single phase field were replaced by two rotating fields of constant magnitude $\frac{\phi_M}{2}$ = 1/2 the maximum value of the single phase field, and each rotating in opposite directions with a velocity of w radians fer second. It can be seen by inspection that these two hypothetical flux vectors are together the vectorial equivalent of the Single phase field (J.2).

It should be kept clearly in mind just what expression (7) represents. It is the voltage in any rotor turn or single conductor coil due solely to the primary flux (not including voltage due to flux which may exist in the axis in quadrature with the Stator poles, and not including that due to leakage flux or resistance drop.) In other woeds, this would be the voltage in a rotor turn consisting of two diametric opposite rotor conductors, if this turn were opencircuited at some point, and no current flowed in this turn nor in anyother rotor conductors.

Now with the normal closed circuit rotor squirrel cage, and rotor current flowing, the rotor currents will produce a cross field or quadrature flux in the X-X axis, (J.3) The evaluation of this cross flux becomes the really complex matter in the analysis. The following factors will be listed to show just why it is so involved. The rotor conductor exciting voltage. This is shown by equation 7. It has 2 frequencies each varying with slip and is different at any instant for each conductor., occupying a difgerent angle .

(2) The rotor conductor current.

There will be a rotor current component for each existing voltage component, the magnitude and phase angle of which will be determined by the rotor impedance for the particular frequency of the component in question.

(3) Angular velocity of Rotor.

The rotor is a moving object, and the angular position of a given conductor will be considerably different at the instant when one voltage component is maximum and the instant when the current due to that voltage is maximum.

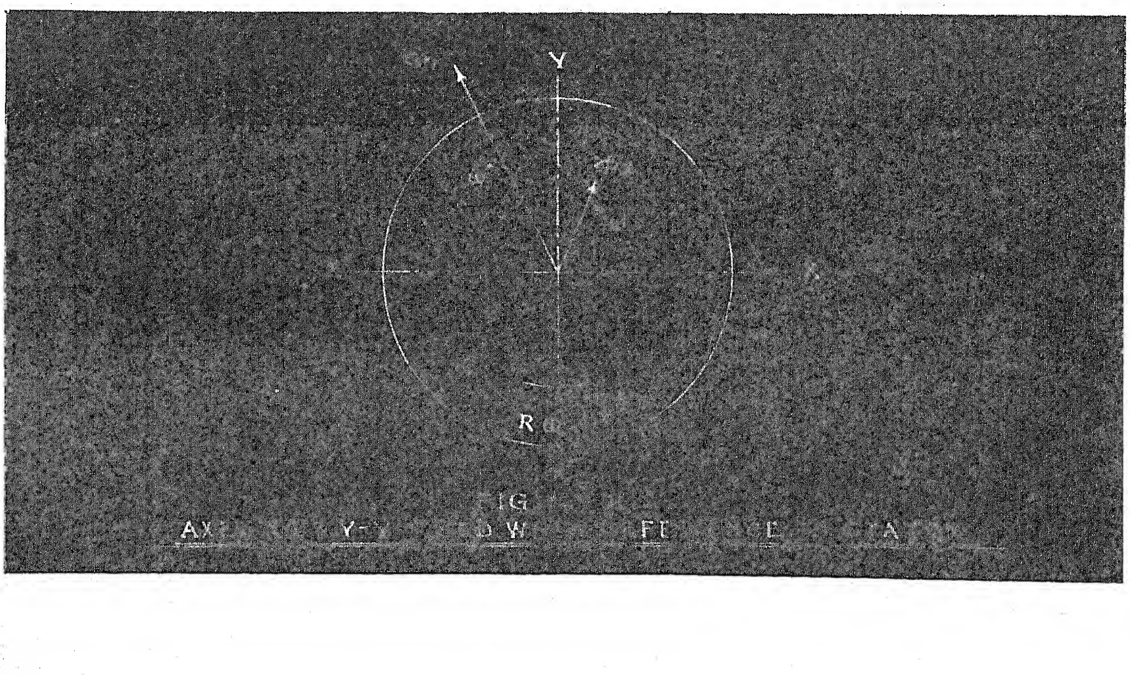
(4) Effect of rotor current on total flux.

The existence of the rotor current results in a complex magnetomotive force producing flux in the cross field axis. The flux may be regarded as unbalancing the relative values of the forward and backward rotating field components, tending toward the production of a pure constant forward rotating field as will be discussed in

detail later.

It is then evident that we have a mechanism involving a number of interdependent variables; and if an attempt is made to be saientificaly precise and to neglect no detail affecting motor performance and to proceed to set for the expressions for the complete rotor currents and voltages all flux values, and final characteristics such as torque at any given slip in terms of motor constants the task becomes extremely formidable even to those familiar with motor theroy in general. (J.3)-

Referring again to expression 7, it is evident that the first component of voltage may be regarded as that due to the backward rotating flux vector (of the revolving field theroy) and the other slip frequency) component is due to the forward rotating field only under open circuit rotor conditions or at standstill, in general, are these 2 flux components equal $\frac{\phi_M}{2}$ Attention is here called to figure 2.



There are then actually 2 voltage components in the conductors of the practical rotor as follows:

$$e_F = \phi_F \; S\,W \; \text{Cos} \; (SWt - \alpha) \quad \text{(Constant omitted)}$$

...(8)

$$e_B = \phi_B \; (2 - S) \; w \; \text{Cos} \left[(2-S) \; wt + \alpha \right]$$

.. (9)

Now at synchronous speed $S = 0$ and $e_F = 0$. It is therefore certain that the cross field which exists at synchronous speed is produced by the current resulting from e_B (J. 4)

<u>Magnetising current in Rotor producing cross field.</u>

The rotor current due to e_B will be $\frac{e_B}{Z_B}$

and lagging behind the voltage e_B by an angle whose cosine is R/Z_B. The in phase component of this current is

$$\frac{\phi_B \; R}{Z_B^2} \; (2-S) \; w \; \text{Cos} \left[(2-S) \; wt + \alpha \right]$$

...(10)

The wattless component of current due to e_B will be

$$\frac{\phi_B \; X_B}{Z_B^2} \; (2-S) \; w \; \text{Sin} \left[(2-S) \; wt + \alpha \right]$$

... (11)

The flux per second square inch in the X-X axis is the product of the magnetomotive force and the permeance per sq. inch. Now if the permeance per sq. inch is constant, each increment of magnetomotive force multiplied by the area it encloses projected in a plane perpendicular to the X-X axis would produce a proportional

increment of flux in that axis. For convenience we may then refer to the product of magnetomotive force and its enclosed area thus projected as a " magnetizing effect"

In developing the equations for rotor voltage and current a coil was considered; this coil lying in a plane which rotated about a line within the plane and perpendicular to the main field flux. In a two pole motor, two rotor bars diametrically opposite with the connecting end rings may be considered as a single turn coil. If the total number of bars on conductors in the rotor is N, there will be a total of N/2 turns (or single turn coils) in the rotor. Since the angular conductor density is $N/2\pi$, if equation 11 is multiplied by $\sin\theta\,(N/2\pi)\,d\alpha$ and integrated from 0 to π, this will include the projected magnetomotive force of every turn in the rotor. The cross field magnetizing effect of the current component given by equation 11 is thus determined as follows.

$$\int_0^{\pi} \frac{\phi_B X_B}{Z_B^2} (2-S)\, w \sin\left[(2-S)\,wt + \alpha\right] \times \sin\theta\, N/2\pi\, d\alpha \qquad \ldots (12)$$

$$= \int_0^{\pi} \frac{\phi_B X_B N (2-S)\, w}{2\pi Z_B^2} \sin\left[(2-S)\,wt + \alpha\right] \times \sin\left[(1-S)\,wt + \alpha\right] d\alpha \qquad \ldots (13)$$

$$= \int_0^{\pi} \frac{\phi_B X_B N (2-S)\, w}{4\pi Z_B^2} \cos wt - \cos\left[(3-2S)\,wt + 2\alpha\right] d\alpha \qquad \ldots (14)$$

For the integral, with α varying from 0 to π

the second factor, or negative quantity in (14) becomes zero, and the magnetizing effect in the X-X axis due to the rotor current of equation 11 is

$$\frac{\phi_B \; X_B \quad N \; (2-S) \; w}{4 \quad Z_B^{\,2}} \quad \text{Cos } wt \qquad \ldots.(15)$$

The corresponding integrated magnetizing effect due to the in phase current component of expression 10 is

$$\frac{- \phi_B N_w \quad (2-S) \; R}{4 \; Z_B^{\,2}} \quad \text{Sin } wt \quad .. \qquad (16)$$

The cross magnetizing effect of this slip frequency rotor current is similarly determined as follows, considering first the reactive component:

The cross magnetizing effect of the slip frequency rotor current is similarly determined as follows; considering first the reactive component:

$$\int_0^{\pi} \frac{\phi_F \; X_F}{Z_F^{\,2}} \; S \; w \quad \text{Sin} \; (\; Swt \; - \alpha) \quad x$$

$$\text{Sin} \left[\; (\; 1-S) \; wt \; + \alpha \right] \frac{N}{2\pi} \; d\alpha \qquad \ldots \; (17)$$

$$= \int_0^{\pi} \frac{\phi_F \; N_{ws} \quad X_F}{4 \pi \; Z_F^{\,2}} \quad \text{Cos} \left[(\; 2S-1) \; wt \; - 2\alpha \right]$$

$$- \text{Cos } wt \quad d\alpha \quad \ldots \quad (18)$$

$$= \quad - \frac{\phi_F \quad N_{ws} \quad X_F}{4 \quad Z_F^{\,2}} \; \text{Cos } wt \quad . \qquad (19)$$

Similarly the magnetizing effect of the slip frequency rotor current in phase with the forward flux is

$$\frac{\phi_F \; N_{ws} \; R}{4 \; Z_F^{\;2}} \; \text{Sin wt} \qquad \ldots (20)$$

The total magnetizing effect or effective rotor magenetomotive force in the X-X axis is then

$$\phi_B \; (F_1' - J F_1) \; - \; \phi_F \; (F_2' - J F_2) \qquad \ldots (21)$$

where

$$F_1 = \frac{N w \; (2-S)^2 \; X}{4 \; \left[R^2 + (2-S)^2 \; X^2 \right] \; 10^8}$$

$$F_1' = \frac{Nw \; (2-S) \; R}{4 \; \left[R^2 + (2-S)^2 \; X^2 \right] 10^8}$$

$$F_2 = \frac{N w \; S^2 \; X}{4 \; R^2 + S^2 \; X^2 \; 10^8}$$

$$F_2' = \frac{N_{ws} \; R}{4 \; (R^2 + S^2 X^2) \; 10^8}$$

(The signs in equations 21 should be changed throughout to be mathematically correct as may be observed. This may be done if it is borne in mind that the J factors represent magnotomotive force components in strict time quadrature with the main flux and really determine the relative direction of the cross field. However equation 21 is written as given to emphasize that the rotor current due to ϕB is primarily responsible for the positive cross field, and the rotor current due to ϕ_F subtracts from it. No damage is done in subsequent developments by these changes in sign. (J.4)

The voltage constant 10^{-8} has been introduced

and the substitutions of $R + J (2-S) X$ for Z_B and $R + JSX$ for Z_F have been made. Also the time phase factors Sin wt and Cos wt are dropped, for the sake of brevity.

The Revolving field concept visualizes 2 rotating flux vectors as shown by figure 2. They add in the Y-Y axis to become indentical with the mutual flux ϕ_M and they subtract in the X-X axis to determine the net cross field flux ϕ_X. The cross field by this assumption is maximum at the instant ϕ_Y is zero. The flux in the 2 cross axes are thus in both space and quadrature. Unquestionably in the practical case there are irregularities or harmonics; and also undoubtedly the flux in the X-X. axis is not precislely zero at the instant that perpendicular to it is maximum. In any theory however ideal concepts are stated in order that practical mathematics may apply in as simple a manner as possible. The practical problem is to evaluate the ideal quantities that the calculated results on that basis are within usual limits of experimental accuracy (J.3)

The rotor may be regarded as an inductive cylinder which tends to maintain constant flux through itself. As it rotates the rotor flux is forced by the primary magnetomotive force to be maximum at the instant ϕy is maximum. The inductive effect of the rotor prevents this flux from dying to zero when the primary flux is zero, except standstill. The minimum rotor flux is in the axis. (If its magnitude were greater than ϕ_M,

ward flux would be negative and no net magnetomotive force would be avilable to produce positivd cross flux.)

The condition at synchronous speed for the flux variation through the rotor is shown by figure 3. Vector FS is the backward rotating flux, and rotates at twice synchronous speed (with respect to the rotor). Vector OF is forward rotating component, and of course at synchronous speed is forced, and constant with respect to rotor. OX is the maximum valud of the cross field flux.

Figüre 4 shows the total flux locus with respect to rotor at a given slip. The locus of total flux traversing the rotor describes a hypotrochoidal curve with respect to it.

In the Y-Y axis the effective rotor ampere turns due to forward and backward rotating currents add instead of subtract as in equation 21; and are balanced by component parts of the primary current. In the X-X axix however, they must mutually approximatdly cancel except for that residual magnetomotive force mecessary for the cross field.

As the forward flux progresses from the Y-Y axis to the X-X axis, the slip frequency rotor current at the Y-Y axis changes from the inphase Value through the maximum to the reactive value. Thus the angle of the flux vector ØF at the instant the forward current is maximum at the Y-Y axis depends on the impedance angle of slip frequency current. The cross field integrated magnetomotive force of the slip frequency current is a periodic reversing quantity; and it is out of phase with that due to the " backward"

current. Since the impedance angles of the two are only equal at zero speed. Figure 5 shows schematically the flux before the flux vectors ØF and ØB at an instant just before the flux through the stator winding is maximum. Conditions (1) from a turn in which the rotor current due to ØF is maximum, and conductors (2) carry maximum rotor current due to ØB. Angle (2) will be greater than angle (1). as the frequency of the current due to $Ø_B$ is greater than that due to $Ø_F$.

The magnetizing effect in the X-X axis in time phase with the primary flux is given by adding expressions 16 and 20. The basic hypothesis used here in assumes that this cross field in phase with the main field is negligible. (J.2)

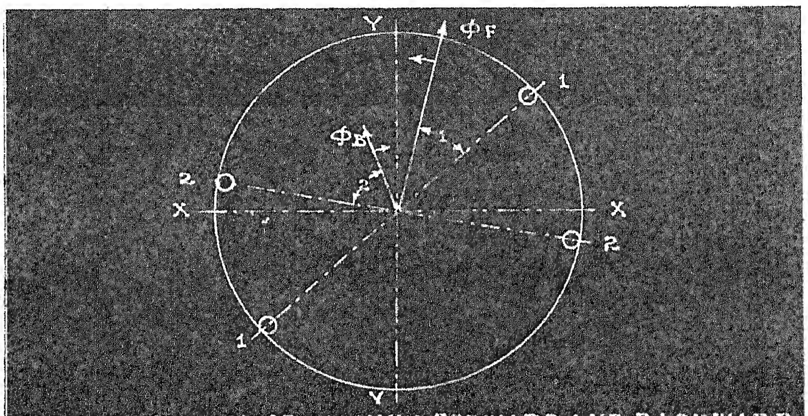


FIG. Nº 5. LOCATION OF MAXIMUM FORWARD AND BACKWARD ROTOR CURRENT AXES FIXED WITH REFERENCE TO STATOR

The magnetizing effect in the X-X axis in phase quadrature with the primary flux is given by adding expressions 15 and 19.

A hypothetical rotor with zero leakage reactance would

be ideal for producing torque in a Polyphase motor. In a single phase stator, such a rotor would not produce a cross field in time quadrature with the main field, but would merely in effect shift the axis of the main field. Hence no motor action would result.

A hypothetical rotor with zero resistance and 90° rotor Impedance angle would be ideal for setting up a cross field in time quadrature, but would produce no torque.

As a general statement then it may be said that the reactive component of rotor current established the rotating field and the inphase component produces torque. The inphase component will however, have an influence on the total flux, which is taken into account in a manner which is sufficiently accurate for practical purposes by the sine factor in equation 23 below.

Determination of Forward and Backward Rotating flux Magnitudes.

The respective values of the forward flux $\emptyset_F$ and the backward flux $\emptyset_B$ are determined as follows. The Sum of the voltages they induce in the stator winding is equal to the line voltage minus primary Impedance drop. (See equation 22).

The difference between these two fluxes is the cross field- (see equation 23)

$$\sqrt{2} \quad (E - i_1 Z_1) = (\emptyset_F + \emptyset_B) \text{ T eff } 10^{-8} \qquad ..(22)$$

where Teff is the total primary turns (for a 2 pole motor) multiplied by the distribution factor. The latter is $\frac{\pi}{4}$

for the ideal case of a winding with sinusoidal distribution. (This corresponds to N/4 or 1/2 the total rotor turns, which are linearly distributed.)

$$(\phi_F - \phi_B) \quad \sin \left[\frac{\tan^{-1} \frac{SX}{R} + \tan^{-1} \frac{(2-S)X}{R}}{2} \right]$$

$$= \phi_B (F_1' - J F_1) \; \phi_F (F_2' - J F_2) \quad x$$

$$\frac{\phi_{Mo}}{\sqrt{2} \text{ im Teff}} \quad \ldots (23)$$

Let $\gamma = \dfrac{\phi_B}{\phi_F}$

From (23)

$$\gamma = \frac{\phi_{Mo} \sqrt{F_2^{12} + F_2^{2}} + \sqrt{2} \text{ im } \text{Teff} \sin E}{\phi_{Mo} \sqrt{F_1^{12} + F_1^{2}} + \sqrt{2} \text{ im } \text{Teff} \sin E} \qquad (24)$$

The primary current i_1 of (22) flowing through the primary turns (effective value) overcomes the rotor magentomotive force by an amount necessary to produce the existing mutual flux.

$$i_1 = \text{im} \frac{\phi_F + \phi_B}{\phi_{Mo}} + \left[\phi_B (F_1' - J F_1) + \right.$$

$$\left. + \phi_F (F_2' - J F_2) \right] \frac{1}{\sqrt{2} \text{ Teff}} \quad \ldots (25)$$

Equation (22)may be modified to the following

$$\frac{E - i_1 Z_1}{E - \text{im} Z_1} = \frac{\phi_F + \phi_B}{\phi_{Mo}} \quad \ldots (26)$$

From (25) and (26)

$$\phi_F = \frac{\phi_{Mo}}{1 + \gamma + \dfrac{Z_1 \phi_{Mo}}{\sqrt{2}\ E\ T_{eff}} \sqrt{(F_1'\gamma + F_2')^2 + (F_1\gamma + F_2)^2}} \qquad ..(27)$$

The treatment of these last equations in algebraic rather than vectorial fashing is an app roximation justified for most practical purposes.

Motor Torque.

For any slip s, we now have the forward rotating flux vector ϕF and the backward Vector ϕB. Assuming that one is not interested in the instantaneous torque (or the pulsating component) the total or the average torque is $T\ \phi F + T\ \phi B$.

The second factor, torque due to the backward rotating flux, is inherently negative and hence total average torque is the difference of these 2 component parts. Each torque component may be calulated as for a polyphase motor where given the flux, slip and rotor constants as follows.

We have the rotor voltages in equations 8 and 9. Whence the inphase rotor current will be ϕF GF Sw Cos (Swt - α) + ϕB GB (2-S) w Cos $\left[(2-S)\, wt + \alpha \right]$ ... (28)

where the conductance

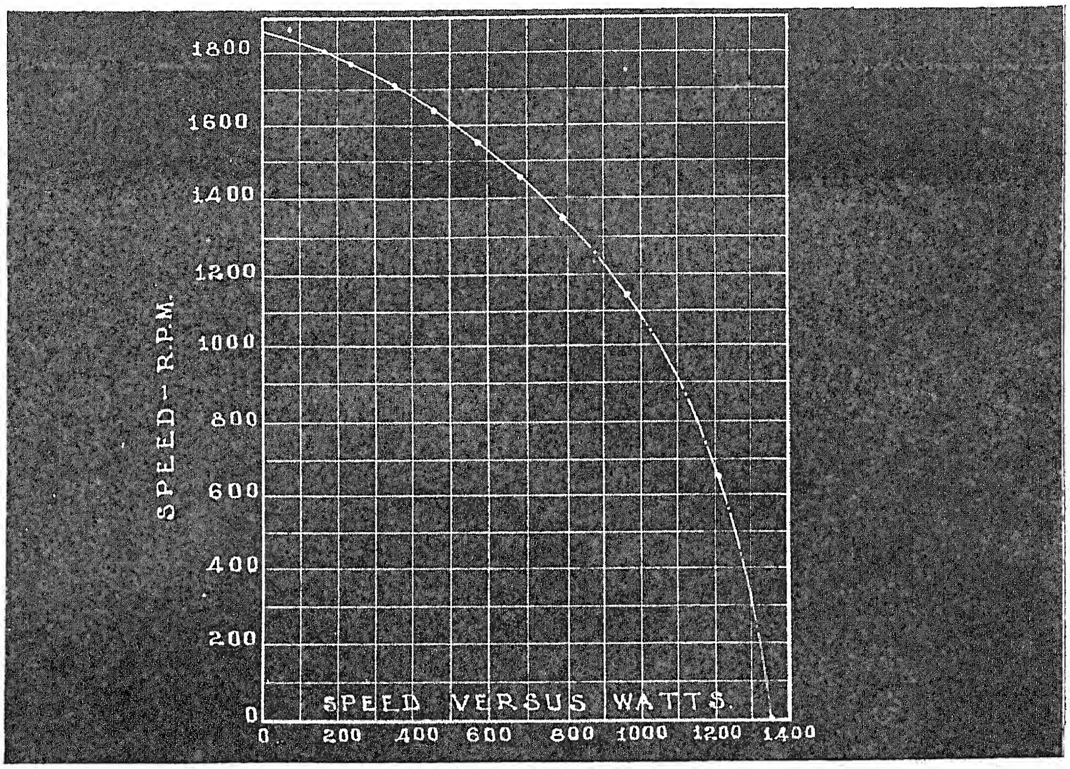
$$G_F = \frac{R}{R^2 + S^2 X^2} \quad \text{and}$$

$$G_B = \frac{R}{R^2 + (2-S)^2 X^2}$$

The inphase component of slip frequency current is maximum at any instant in the conductor which is at the point of the air gap carrying maximum (forward) flux. Both the forward flux density and the inphase slip frequency current magnitude will vary around the air gap at any instant as cos δ ; δ being zero where ϕF is maximum.

The forward flux per unit length of air gap (l = density x length of rotor) at the point where ϕF is maximum is $\phi F/d$. where d is the diameter of the rotor. The torque being a tangential force (whibh is product of flux density and current) multiplied by the lever arm d/2. then the torque due to 2 rotor conductors forming a turn at the diametrically opposite points of maximum flux density will be found by multiplying the in-phase current by the total forward flux ϕF. In symbols the above may be stated in the abbrevated form $T = \frac{\phi F}{d} \times 1 \times \frac{d}{2} \times 2 = \phi F \times 1.$

for the 2 conductors or one turn)



The quantity I is the in phase " forward" current in the rotor bar in which it is maximum at any instant.

The torque due to ØF for one rotor turn then is

$$\text{Ø}F^2 \, G \, F \, Sw \, \text{Cos}^2 \delta = \frac{\text{Ø}F^2 G \, F \, Sw}{2} \left(1 + \text{Cos } 2\delta \right) \quad \text{. . (29)}$$

Multiplying by $\frac{N \, 10^{-8}}{2\pi} \, d\delta$ and integrating from $\delta = 0$ to $\delta = \pi$

$$T \, \text{Ø}F = \frac{\text{Ø}F^2 \, Nsw \, R}{4 \left(R^2 + S^2 X^2 \right) 10^8} = F_2' \, \text{Ø}F^2 \quad \text{..(30)}$$

Similarly the torque due to ØB is

$$T \, \text{Ø}B = F_1' \, \text{Ø}B^2 \quad \text{...} \quad (31)$$

Total average torque is

$K \; (F_2' \, \phi F^2 \; - \; F_1 \, \phi B^2)$ ... (32)

K is a constant which has the value 0.7376×10^{-8} to reduce to pound feet.

For a motor having more than two poles, expression 32 will be modified.

Conclusions.

Present methods of calculating the performance of single phase Induction Motors are quite tedious, and probably do not convey to the mind of the practical engineer a logical concept of the operation of the motor such as is the case with generally used methods of Polyphase motor design calculation. It seems desirable to have a unified and co-ordinated theory for single phase Induction motors, including formulas which are simplified as much as possible. (with justifiable approximations) and which convey unabstruse concepts. The formulas developed may be used as the basis of a routine calculation Procedure.

An advantage of the formulas of this type is that they deal with real values - actual rotor current in a rotor bar in amperes, and so on. The value of rotor impedance does not change in using the same motor with 2 different stator windings for example, as it does when equivalent primary values are used. Flux densities in the cross field axis and other similar values are explicitly evident.

In applying the equations developed it is necessary first to determine motor constants by established methods.(J.5)

CHAPTER VI.

Symmetrical Components as Applied to the Single-Phase Induction Motor

Assumptions:

1. The motor is magnetically symmetrical

(a) Sinusoidal distribution of conductors in the stator slots

(b) Uniform air gap

(c) Uniform iron permeance.

2. Sinusoidal voltage applied.

3. Linear constants - Saturation of iron flux paths is negligible.

4. Friction and windage, iron loss, and load losses are subtractive from the rotor output.

5. Main and start phases are in space quadrature.

General Considerations.

The solution of power or current flow in polyphase circuits when the applied voltages are not balanced, or when the load impedances are not equal can be accomplished best by the application of the methods of symmetrical components (J.1) We will show presently that the case of the single phase Motor is a special case of an unbalanced two phase motor.

Let us assume the general two phase circuit with phase voltages (V_m and V_s) unequal in magnitude and not in time quadrature. It has been shown that two ~~xx~~ balanced systems can be formed (J.4) one system, called the "Positive Sequence" (+) consists in magnitude and in time quadrature and have (+) phase rotation or the same rotation

as the V_M and V_s system; the other system called the " Negative sequence" (-) consists of other components of V_M and V_s equal in magnitude and in time quadrature, but phase rotation is opposite that of the positive sequence componetns. No " Uniphase" or zero-sequence" components exist.

Part I.

By definition we then have

$$V_M = V_M^+ + V_M^- \qquad (1)$$

$$V_S = V_S^+ + V_S^- \qquad (2)$$

$$V_s^+ = V_M + 90^o = + j V_M^+ \qquad (3)$$

$$V_M^- = V_M^- / 90^o = - j V_M^- \qquad (4)$$

Phase s leads phase M in time angle. From equations 2, 3,, and 4 we then have

$$V_s = j (V_M^+ - V_M^-) \qquad (2a)$$

which defines V_s in terms of the components of V_M

When the magnitude and time phase of V_M and V_S are each known are each known, the symmetrical components of each can be calculated by adding equation 1 and - g times equation 2 a and clearing.

$$V_M^+ = \frac{V_M - j V_S}{2} \qquad (5)$$

By subtracting - g times equation 2 a from equation 1 and clearing we have

$$V_M^- = \frac{V_M + j V_S}{2} \qquad (6)$$

All the foregoing quantities are vectors representing rms

magnitude and time phase voltages.

By the same procedure we arrive at current values

$$I_M = I_M^+ + I_M^- \qquad ..(7)$$

and

$$I_S = I_S^+ + I_S^- \qquad ..(8)$$

$$I_S^+ = J\, I_M^+ \qquad (9)$$

$$I_S^+ = - J\, I_M^- \qquad (10)$$

$$I_M^+ = \frac{I_M - J\, I_S}{2} \qquad (11)$$

and

$$I_M^- = \frac{I_M + J\, I_S}{2} \qquad (12)$$

In a general two phase system as shown in figure, 1, Z_M and Z_S are the load impedances of phases M and S respectively. Now applying Ohm's law, we have (J.1)

$$V_M = I_M\, Z_M \qquad (13)$$

and

$$V_S = I_S Z_S \qquad (14)$$

Then from equations 13, 7 and 1

$$V_M^+ + V_M^- = (I_M^+ + I_M^-)\, Z_M \qquad (15)$$

and from equations 14, 8, and 2

$$V_S^+ + V_S^- = (I_S^+ + I_S^-)\, Z_S \qquad (16)$$

or from equations 3 and 4 equation 16 becomes

$$J\, V_M^+ - J\, V_M^- = (+ J\, I_M^+ - J\, I_M^-)\, Z_S \qquad (16a)$$

or

$$V_M^+ - V_M^- = (I_M^+ - I_M^-)\, Z_S \qquad (16b)$$

Adding equations 15 and 16 b we have

$$V_M^+ = \frac{1}{2} \left[I_M^+ (Z_M + Z_S) + I_M^- (Z_M - Z_S) \right] \quad ..(17)$$

subtracting equation 16b from equation 15 and clearing, we have

$$V_M^- = \frac{1}{2} \left[I_M + Z_M - Z_S) + I_M^- (Z_M + Z_S) \right] \quad ..(18)$$

In the case of a transformer or single phase motor where there are not the same turns in each of the windings or phases then, (J. 1)

$$I_S^+ = \frac{M}{S} I_M^+ = \frac{I_M^+}{a} \quad ...(19)$$

and $$I_S^- = \frac{I_M^-}{a} \quad ..(20)$$

$$V_S^+ = \frac{S}{M} V_M^+ = a V_M^+ \quad .. (21)$$

$$V_S^- = a V_M^-$$

where

M = effective conductors of the M Phase

S = effective conductors of the S phase.

$$a = \frac{S}{M}$$

we can now write

$$V_S^+ = J a V_M^+ \quad (3a)$$

$$V_S^- = - J a V_M^- \quad (4a)$$

$$V_S = J a (V_M^+ - V_M^-) \quad (2b)$$

Similarly

$$I_S = \frac{J}{a} (I_M^+ - I_M^-) \quad (8a)$$

In a single phase motor it is obvious that

$$V_S = V_M \quad (23)$$

Now equation 2b becomes

$$V_M = Ja \left(V_M^+ - V_M^- \right) \qquad .(2c)$$

Further more we will find that the impedance to positive sequence currents is not the same as for negative sequence currents in rotating apparatus. (J.2)

Let Z_M^+ = Impedance to I_M^+

Z_M^- = Impedance to I_M^-

Z_S^+ = Impedance to I_S^+

Z_S^- = Impedance to I_S^-

Multiplying equation 2c by $-J/a$ and adding to equation 1, we obtain

$$V_M^+ = \frac{V_M}{2} \left(1 - \frac{J}{a} \right) \qquad .. (24)$$

or by subtarction we obtain

$$V_M^- = \frac{V_M}{2} \left(1 + \frac{J}{a} \right) \qquad (25)$$

From equation 3a and 24 we have

$$V_S^+ = \frac{V_M}{2} \left(1 + Ja \right) \qquad (26)$$

and from equation 4a a nd 25 we have

$$V_S^- = \frac{V_M}{2} \left(1 - Ja \right) \qquad (27)$$

Equation 13 and 14 now can be reconsidered to gether with equations 9, 10, 8a and 23 and at the same time the sequence impedeances Z_M^+, Z_M^- Z_S^* and Z_S^- can be included

$$V_M = I_M^+ Z_M^+ + I_M^- Z_M^- \qquad (13a)$$

$$V_M = \frac{J}{a} \left(I_M^+ Z_S^+ - I_M^- Z_S^- \right) \qquad (14a)$$

To solve for I_M^- multiply equation (13a) by $\frac{Z_S}{a^2}$

and equation 14 a by $\dfrac{-J}{a\, Z_M^{+}}$

$$V_M \frac{Z_S^+}{a^2} = I_M^+ \; Z_M^+ \frac{Z_S^+}{a^2} + I_M^- \; Z_M^- \frac{Z_S^+}{a^2} \qquad \text{..(13b)}$$

$$-J\, V_M \frac{Z_M^+}{a} = I_M + \frac{Z_S^-}{a^2} Z_M^+ - I_M^- \, Z_M^* \frac{Z_S^-}{a^2} \qquad \text{..(14b)}$$

If we subtract equations 14 b from equation 13 b

$$V_M \left[\frac{Z_S^+}{a^2} + J \frac{Z_M^+}{a} \right] = I_M^- \times \left[\frac{Z_M^- \; Z_S^+}{a^2} + \frac{Z_M^+ \; Z_S^-}{a^2} \right]$$

or

$$I_M^- = V_M \left[\frac{Z_S^+ + J\, a\, Z_M^+}{Z_M^+ \, Z_S^- + Z_M^- \, Z_S^+} \right] \qquad (28)$$

By applying equation 10 and 2)

$$I_S^- = V_M \left[\frac{Z_M^+ - J \dfrac{Z_S^+}{a}}{Z_M^* \, Z_S^- + Z_M^- Z_S^+} \right] \qquad (29)$$

To solve for I_M^+ Multiply equation 13 a by $\dfrac{Z_S^-}{a^2}$ and equation 14 a by $J/a\, Z_M^-$

$$V_M \frac{Z_S^-}{a^2} = I_M^+ \; Z_M^+ \frac{Z_S^-}{a^2} + I_M^- \, Z_M^- \frac{Z_S^-}{a^2} \qquad \text{..(13c)}$$

$$J\, V_M \frac{Z_M^-}{a} = -\, I_M^+ \, Z_S^+ + \frac{Z_M^-}{a^2} + I_M^- Z_S^- \frac{Z_M^-}{a^2} \qquad (14c)$$

If we subtract equation 14 c from equation 13c

$$V_M \left[\frac{Z_S^-}{a^2} - J \frac{Z_M^-}{a} \right] = I_M^+ \times \left[Z_M^+ \frac{Z_S^-}{a^2} + Z_M^- \frac{Z_S^+}{a^2} \right]$$

or

$$I_M^+ = V_M \left[\frac{Z_S^- - J\,a\, Z_M^-}{Z_M^+ Z_S^- + Z_M^- Z_S^+} \right] \quad ..(30)$$

By applying equations 9 and 19

$$I_S^+ = V_M \left[\frac{Z_M^- + J \dfrac{Z_S^-}{a}}{Z_M^+ Z_S^- + Z_M^- Z_S^+} \right] \quad ..(31)$$

From equation 7, 28, and 30 we then obtain the general equations,

$$I_M = V_M \left[\frac{Z_S^+ + Z_S^- + J\,a\,(Z_M^+ - Z_M^-)}{Z_M^+ Z_S^- + Z_M^- Z_S^+} \right] \quad (32)$$

and from equations 8, 29, and 31 we then obtain

$$I_S = V_M \left[\frac{(Z_M^+ + Z_M^-) - \dfrac{J}{a}(Z_S^+ - Z_S^-)}{Z_M + Z_S^- + Z_M^- Z_S^+} \right] \quad (33)$$

The line current is the combined current of main and start phases.

$$I_L = I_M + I_S = V_M (Z_M^+ + Z_M^- + Z_S^+ + Z_S^-) + \frac{J\left(-a Z_M^- \dfrac{Z_S^-}{a} - \dfrac{Z_S^+}{a}\right)}{Z_M^+ Z_S^- + Z_M^- Z_S^+} \quad ...(34)$$

Part II.

These general equations will now be applied specifically to the single-phase motor.

The equivalent circuits of the M phase are shown in Fig. 2.

$$Z_{2M}^{+} = \frac{R_2}{S} + J X_2 \qquad (35)$$

Secondary impedance to positive sequence current.

By combinging the magnetising imødence $J^{X\emptyset}$ with the secondary Impedance Z_{2M}^{+} the apparent secondary Impedance can be written (J.3)

$$Z_2^{+\prime} = \frac{1}{\dfrac{1}{R_2/S + J X_2} + \dfrac{1}{J X_\emptyset}}$$

$$= \frac{X_\emptyset^{\,2} \; R_2 / S}{\left(\dfrac{R_2}{S}\right)^2 + (X_2 + X_\emptyset)^2} +$$

$$\frac{J X_\emptyset \left(\dfrac{R_2}{S}\right)^2 + X_\emptyset X_2 (X_\emptyset + X_2)}{\left(\dfrac{R_2}{S}\right)^2 + (X_\emptyset + X_2)^2} \qquad ..(36)$$

Or,

$$Z_2^{+\prime} = R_2^{+\prime} + J X_2^{+\prime} \qquad (36a)$$

where $R_2^{+\prime}$ and $J X_2^{+\prime}$ are apparent

Secondary positive sequence resistance and apparent secondary positive sequence reactance respectively.

Similarly,

$$Z_{2M}^{-} = \frac{R_2}{2-S} + J X_x \qquad (37)$$

Secondary impedance to negative sequence current and (J3)

$$Z_2^{-1} = \frac{X_\phi^2 \; \dfrac{R_2}{2-S}}{\left(\dfrac{R_2}{2-S}\right)^2 + (X_\phi + X_2)^2} +$$

$$J\,\frac{X_\phi \left(\dfrac{R_2}{2-S}\right)^2 + X_\phi \, X_2 (X_\phi + X_2)}{\left(\dfrac{R_2}{2-S}\right)^2 + (X_\phi + X_2)^2} \qquad (38)$$

$$Z_2^{-1} = R_2^{-1} + J\,X_2^{-1} \qquad ..(38a)$$

Where R_2^{-1} and X_2^{-1} are apparent secondary negative sequence resistance and apparent secondary negative sequence reactance respectively.

The primary impedance of the M phase is

$$Z_{1M}^{+} = Z_{1M}^{-} = R_{1M} + J\,X_{1M} \qquad (39)$$

and of the S phase is

$$Z_{1S}^{+} = Z_{1S}^{-} = (R_{1S} + R_c) + J\,(X_{1S} - X_c) \qquad ..(40)$$

in the case of a resistance split phase motor having external Resistance R_c.

The generalized impedances of the Single phase motor can now be written : (J.3)

$$Z_M^{+} = R_{1M} + R_2^{+1}) + J\,(X_{1M} + X_2^{+1}) \qquad (41)$$

$$Z_M^{-} = R_{1M} + R_2^{-1}) + J\,(X_{1M} + X_2^{-1}) \qquad (42)$$

$$Z_S^{+} = (R_{1S} + R_c + a^2 R_2^{+1}) +$$

$$J\,(X_{1S} - X_c + a^2 X_2^{+1}) \qquad ..(43)$$

$$Z_X^- = (R_{1S} + R_c + a^2 R_2^{-1}) + j\,(X_{1S} - X_c + a^2 X_2^{-1}) \quad ..(44)$$

If we combine equation 30 with equations 41, 42, 43, 44,

$$I_M^+ = V_M \times \frac{(R_{1S} + R_c + a^2 R_2^{-1}) + j\,(X_{1S} - X_c + a^2 X_2^{-1}) - ja\,(R_{1M} + R_2)^{-1} + a\,(X_{1M} + X_2^{-1})}{\begin{gathered}\left[(R_{1M} + R_2^{+1}) + j\,(X_{1M} + X_2^{+1})\right]\left[R_{1S} + R_c + a^2 R_2^{-1} + j\,(X_{1S} - X_c + a^2 X_2^{-1})\right] + \\ \left[(R_{1M} + R_2^{-1}) + j\,(X_{1M} + X_2^{-1})\right]\left[(R_{1S} + R_c + a^2 R_2)^{+1} + j\,(X_{1S} - X_c + a^2 X_2)^{+1}\right]\end{gathered}} \quad ...(45)$$

$$= V_M \times \frac{\left[(R_{1S} + R_c + a^2 R_2^{-1}) + a\,(X_{1M} + X_2^{-1})\right] + j\,(X_{1S} - X_c + a^2 X_2)^{-1} - a\,(R_{1M} + R_2^{+1})}{\text{Denominator of equation 45.}}$$

Likewise from equation 28,

$$I_M^- = V_M \times \frac{\left[(R_{1S} + R_c + a^2 R_2^{+1}) - a\,(X_{1M} + X_2^{+1})\right] + j\left[(X_{1S} - X_c + a^2 X_2^{+1}) + a\,(R_M + R_2^{+1})\right]}{(\text{Denominator of equation 45})} \quad ....(46)$$

and from equation 32,

$$I_M^{+} = V_M \times (2 R_{1S} + 2 R_c + a^2 (R_2^{+1} + R_2^{-1}) +$$

$$J (2 X_{1S} - 2 X_c + a^2 (X_2^{+1} + X_2^{-1})$$

$$\frac{+ Ja (R_2^{+1} - R_2^{-1}) + J (X_2^{+1} - X_2^{-1})}{\text{(Denominator of equation 45)}} \qquad ..(47)$$

From equation 31,

$$I_S^{+} = V_M (R_{1M} + R_2^{-1}) - \frac{1}{a} (X_{1S} - X_c + a^2 X_2^{-1})$$

$$+ J (X_{1M} + X_2^{-1}) + \frac{1}{a} (R_{1S} + R_c +$$

$$\frac{+ a^2 R_2^{-1})}{\text{(Denominator of Equation 45)}}$$

..(48)

From equation 29,

$$I_S^{-} = V_M \times (R_{1M} + R_2^{+1}) + \frac{1}{a} (X_{1S} X_c + a^2 X_2^{+1})$$

$$\frac{+ J (X_{1M} + X_2^{+1}) - \frac{1}{a} (R_{1S} + R_c + a^2 R_2^{+1})}{\text{(Denominator of equation 45.)}}$$

..(49)

From equation 33,

$$I_S = V_M \times (2 R_{1M} + R_2^{+1} + R_2^{-1}) + J (2 X_{1M} + X_2^{+1} + X_2^{-1})$$

$$\frac{- \frac{J}{a} (a^2 (R_2^{+1} - R_2^{-1}) + Ja^2 (X_2^{+1} - X_2^{-1})}{\text{(Denominator of equation 45)}}$$

...(50)

Output and Torque.

In the case of a balanced two phase motor, the gross rotor output is

$$WO_{2\emptyset} = 2 \quad I_2{}^2 \frac{R2}{S} (1 - S) \quad \text{watts.} \quad (51)$$

Here only positive sequence currents exist. The net rotor output is the gross output less friction and windage, iron loss, and load loss.

The motor torque in ounce feet is

$$T_{2\emptyset/} = W\,O_{net} \times \frac{16 \times 33{,}000}{2\,\Pi \times r.p.m. \times 746}$$

$$= \frac{112.7}{r.p.m} \times W\,O_{net}$$

The positive sequence currents in the secondary winding produce Torque in the direction of the phase rotation of these currents. When negative sequence currents exist they produce a counter torque. As we are dealing with two balanced systems of two phase currents we may consider the torques they produce as positive sequence and negative sequence (J.3)

Since,

$$I_2{}^+{}_M = \frac{J\,X_\emptyset}{R_2/S + J(X_\emptyset + X_2)} \; I_M{}^+ \qquad ..(53)$$

or

$$I_{2M}^{+} = \frac{X_\emptyset (X_\emptyset + X_2) + J\,X_\emptyset \frac{R2}{S}}{\left(\frac{R_2}{S}\right)^2 + (X_\emptyset + X_2)^2} \; I_M^{+} \qquad ..(53a)$$

and

$$I_{2M}^{-} = \frac{J X_\phi}{\frac{R_2}{2-S} + J(X_\phi + X_2)} \; I_M^{-} \qquad ..(54)$$

$$= \frac{X_\phi (X_\phi + X_2) + J X_\phi \frac{R_2}{2-S}}{\left(\frac{R_2}{2-S}\right)^2 + (X_\phi + X_2)^2} \; I_M^{-} \qquad ..(54\ a)$$

We can now write the gross power output in terms of the component secondary currents.

$$WO \text{ gross} = \left[I_{2M}^{+2} \frac{R_2}{S} + I_{2S}^{+2} a^2 \frac{R_2}{S} - I_{2M}^{-2} \frac{R_2}{2-S} - I_{2S}^{-2} a^2 \frac{R_2}{2-S} \right] (1 - S) \qquad ..(55)$$

From equations 9 and 10

$$I_{2S}^{+} = \frac{I_{2M}^{+}}{a} \qquad \text{and}$$

$$I_{2S}^{-} = \frac{I_{2M}^{-}}{a}$$

Substituting these in equation 55, we have

$$WO \text{ gross} = 2 \left[I_{2M}^{+2} \frac{R_2}{S} - I_{2M}^{-2} \frac{R_2}{2-S} \right] \times (1-S) \text{ watts} \qquad ...(55a)$$

Now observe that from equations 53a, 36 and 36a

$$I_{2M}^{+2} \frac{R_2}{S} = \frac{X_\phi^2 \frac{R_2}{S} \; I_M^{+2}}{(R_2/S)^2 + (X_\phi + X_2)^2}$$

$$\text{ɫ} = R_2^{+1} \; I_M^{+\,2} \qquad \ldots (56)$$

and from equations 54a, 38 and 38a

$$I_{2M}^{-\;2} \frac{R_2}{2-S} = \frac{X_\emptyset^{2} \; \frac{R_2}{2-S}}{\left(\frac{R_2}{2-S}\right)^2 + (X_\emptyset + X_2)^2} \; I_M^{-} \,/^{2}$$

$$= R_2^{-1} \; I_M^{-\;2} \qquad \ldots (57)$$

so that the equation 55a reduces to the more conveninet form:

$$WO\ \text{gross} = 2 \quad I_M^{+\,2} R_2^{+1} - I_M^{-\;2} \; R_2^{-1} \quad (1 - S)\text{watts}$$

,,(55b)

<u>Efficiency and power factor</u>.

By adding I_M (euquation 47) and I_S (equation 50) the line current I_L can be obtained.

The " in phase" component of IL multiplied by the line voltage (VM) gives us the watts input, while IL x VM gives us the volt-ampere input.

$$\text{Power Factor} = \frac{\text{Watts Input}}{\text{Volt-amperes input}} \qquad ..(58)$$

After subtaracting the rotational losses (friction and windage iron loss and load loss) from equation 55b, we have the net output watts

WO nett

$$\text{Efficiency} = \frac{\text{WO Net}}{\text{Watts Input.}} \qquad ..(59)$$

CHAPTER VII.

NOISE.

7.1 Magnetic humming.

First we have magnetic humming, which is due to vibration of the steel laminations caused by the rapid pulsation of flux in the teeth of the stator and rotor. When the rotor teeth are opposite stator teeth, the magnetic reluctance, is a minimum; when the rotor slots are opposite stator teeth we have maximum reluctance. Thus, we have the flux varying rapidly in stator and rotor teeth as this rotor revolves. Since the pull on the teeth varies as the square of the flux density, it is clear they are subjected to a succession of rapidly varying mechanical impulses which cause vibration of the laminations. The frequencies of these vibrations are in the region to which the ear is most sensitive (J.7)

The design of the teeth in both stator and rotor and especially the design of the lip of the teeth, with semi-closed slots is all important. They must be sufficiently rigid to withstand the forces to which they are subjected, without vibration and obviously the cores must be very firmly held in position. In this case skewing of the slots is very helpful in such noise elimination.

If the very quiet motors are required, one must not only use skewing of slots, but the normal flux density in the teeth and gap must be kept low, and the air gap must be moderately

large to cut down the influence of parasitic fluxes and torques. Extreme quietness means, in effect, a very liberal design of motor (7.7).

7.2 Noise Due to Windage.

As the rotor revolves air currents are set up and a puff of air will pass through the stator, between the stator coils, every time a rotor tooth comes opposite a stator tooth so that the machine acts as a siren. The intensity of the note emitted depends on the peripheral velocity of rotor. (V' per minute) while its frequency or pitch p.

p = the number of puffs per second

= the number of rotor slots x revolutions per second

$$= \frac{V}{5} \cdot \frac{1}{\text{Rotor slot pitch in inches}}$$

The higher the pitch of the note the more objectionable it becomes, and an note with a frequency greater than 1560 c/s which is the high G of a Soprano, is very objectionable if loud and long sustained. For a velocity of 8000 ft. per minute and a rotor slot pitch of 1", the frequency of the windage note is 1600 c/s.

Noise due to windage can be lowered in intensity by blocking up the rotor vent ducts; if the motor then runs hot due to poor ventilation some other method of cooling must be adopted, such as blowing air across the external surface of the punchings, or the motor may be totally enclosed and cooled by forced ventilation. The intensity of the note can be

greatly reduced by staggering the vent ducts and since the air gap clearance is large in high speed machines, which are the only ones that are noisy due to windage the ventilations of such machines will not be seriously effected. It should also be noted that for high speed machines a large number of narrow ducts will give quieter operations than a small number of wide ducts because the velocity of air in each duct will be reduced (B.1)

7.3 Noise due to Pulsation of air gap flux.

The flux in the rotor tooth pulsates from a maximum when the rotor tooth is opposite stator tooth and to a minimum when the rotor tooth is opposite to a stator slot and the frequency of this pulsation is equal to the number of stator teeth. x revolutions per second. This pulsation of flux causes a noise which varies in intensity with the voltage. To minimise the noise, the machine should be designed to have a minimum pulsation loss, the condition for that is that the rotor teeth shall be equal to the stator slot pitch and the rotor slot shall be narrow. The noise due to pulsation of main flux may be minimised by stacking the rotor stampings slightly. ' J.4)

7.4 Noise due to leakage flux.

The principal cause of noise in induction motors is the variation in the reluctance of the Zig Zag leakage path. When the stator tooth is opposite to that of rotor

tooth the zig zag leakage flux is a minimum and when rotor tooth is opposite to that of stator slot the flux is a maximum, so that there is a pulsation of flux in the tooth tips and two nodes are emitted. To prevent this noise, the variation in the zig zag leakage is to be reduced to a minimum and the most satisfactory way to do this is to make the zig zag leakage as small as possible. This leakage flux is proportional to the ampere conductors per slot and inversely proportional to air gap clearance, and it has been found from experience that in order to prevent excessive noise upto 25 percent overload the ratio of ampere conductors per slot at full load to the airgap clearance should not be greater than 14×10^3 for machines without open stator and partially closed rotor slots, or 12×10^3 for machines with partially closed slots for both stator and rotor.

f_1 = frequency = number of rotor slots x revolutions per second.

f_2 = Number of stator slots x revolutions per second.

It is found that as f_1 and f_2 approach each other, the noise becomes more and more objectionable and that for even lower values of the above stated ratio the noise will be objectionable if the number of rotor slots differ with in 20 percent of stator slots from that of stator slots (B.1)

7.5 Measurement of Noise.

Measurement of the total noise of the small motor undoubtedly presents a different aspect from the standpoint of the practical engineer. The quietness of

the modern fractional Horse power motors renders this problem more difficult. From our present discussion Noise includes all sound undesired by recipient. It is a combination of various notes of different amplitude with no regular phase relationship. Noise is measured in phons. It is intensity level in decibels of a 1000 cps note judge by listeners to be equally loud as a noise. If the noise is also of 1000 cps; the loudness in phons is equal to the intensity in decibels, the sound intensity in decibels being 10 times the logarithm to base 10 of the ratio of intensities. To calculate sound level at a point;

1. Determine amplitude and mode of vibration of body.
2. Calculate sound intensity produced at surface of body.
3. Calculate sound intensity at the distant point from radiation pattern.
4. Find sound level from Fletchers graph.

In describing the performance of the Secondary noise meters, King and Churcher (J. 8) has described a subjective noise meter in which the observer listens simultaneously to the noise with one ear and a reference tone of adjustable intensity presented to the other ear by a Telephone Receiver. & This apparatus can be used only to measure the total noise. The reference tone is then adjusted by trial to what he judges to be equally loud with the noise. In a two telephone type meter alternative listening is made possible by the use of the two receivers fitted with soft rubber caps so that when the receivers are placed on the ear the reference tone is

heard and the noise under measurement is substantially excluded. The criterion of loudness equality of noise and reference tone is that when the receivers are quickly removed or applied no change of loudness is detectable. Since the result mainly depends on such judgement, for an important test to have less than 5 observers is inadvisable and that to repeat the test once or twice and take the mean of all readings is useful in minimising the effect of casual divergences. This type of meter is better than the former because,

1. the two telephone meter avoid the abnormal condition of different sounds in the two ears heard simultaneously
2. The criterion of balance is not loudness equality but absence of change of loudness where the sounds are heard alternatively, which is likely to be more discriminating.
3. Binaural listening gives greater sensitivity to loudness differences than monaural listening.
4. The mode of listening of later is the nearest practicable approximation to that embodied in the definition of equal loudness.

In objective meters noise is measured by purely instrumental means such that any personal observational error was quite negligible. It is easy to adjust the sensitivity of an objective meter by means of weighting circuits so that it indicates correctly for pure tones in accordance with accepted equal loudness relation. In addition such meters have been used to measure complex noises and the implied assumptions as to the law of summation of the human ear for such

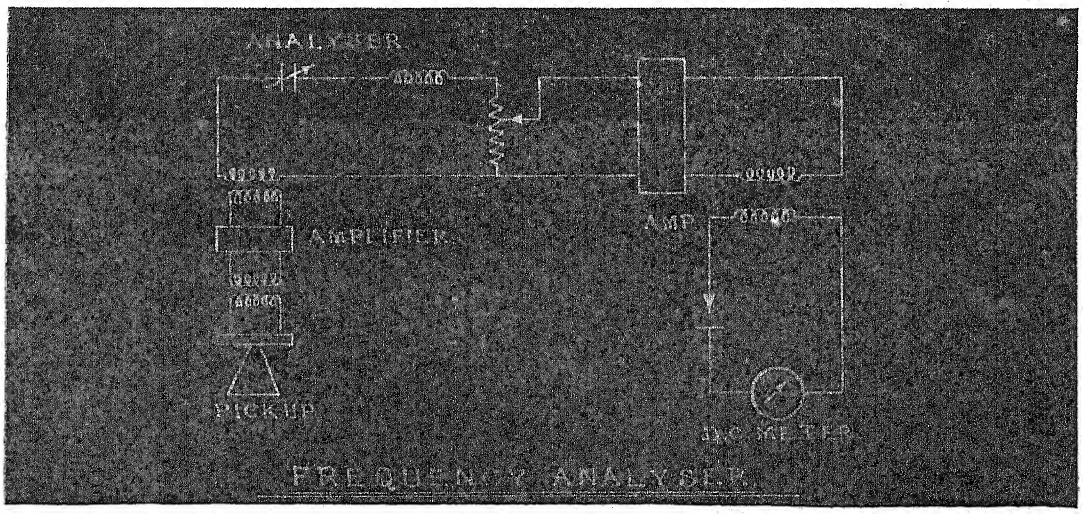
noises call for verification. Hence copper oxide rectifier type and thermal type instruments have been used under the assumption that energy summation of the weighted components is correct.

Analysers using the principle of resonance are quite common. In a set up that was tried (study of noise in Electrical apparatus by Spooner, AIEE Vol 48, July '29 P. 747) (J.2) a Electro dynamic microphone was used that feeds into a audiofrequency transformer, amplifier and stepdown transformer. Although a condenser microphone has a stable and constant response over a wide range of frequencies, its sensitivity is poor. The crystal microphone gives same results as condenser microphone except where low frequency components are predominent and in which case the readings are lower. However moving coil microphone gives erroneous readings due to stray magnetic fields. The voltage across the non-Inductive resistance with a variable tap is fed into a second amplifier, stepdown transformer copper oxide rectifier, and a D.C. Milliameter. A reading will be shown on the D.C. meter only if a sound having a compenent corresponding to the setting of the resonent circuit were to impinge on the microphone.

For calibration the system is arranged as shown. The constants of the Electrodynamic Analyser pick up must be known. The standard microphone is connected to a milliameter through amplifier and is calibrated between input volts and milliammeter readings. A sine wave of known frequency

is applied to loud speaker and after tuning for resonance condenser setting and meter readings are noted. This gives an amplitude and frequency on the calibration curve of Analyser. The test is hence repeated for various frequencies and amplitudes.

Soud pressure measurements were made on apparatus like integrating meters and transformers for domestic use. But sound measuring apparatus used in conjunction with machinery must be sensitive to detect small intensities as well as largest intensities. Regarding the accuracy of the instruments the relative accuracy of the measurements for ordinary intensities need not be more than 5% and of frequency measurement need not be more than 1%. The insrument must be capable of reasonably rapid operation and also of measuring transient as well as steady sounds. The apparatus must be capable of operating even on a bench where there is a good amount of vibration and Electrical disturbance.



FREQUENCY ANALYSER

For studying transients in sound as well as for the knowledge of wave shape oscillograms are used. But analysis of thesewave forms into their components by graphical method is a loborious process or sometimes impracticable. There are many simple methods of measuring frequencies but they give no information in regard to intensity. For obtaining a rough measure of the relative sound intensity is to couple a telephone transmitter the receiver being placed in a sound proof chamber. The later is then shunted using known resistances so that the sound is just audible. From the impedence of the receiver and the shunting resistance the relative intensities can be got. Non Electrical apparatus such as those using mechanical resonance are not very satisfactory.

In the Electrical method is is necessary to use a microphone where the R.M.S. electromotive force generated per unit R.M.S. Sound pressure is independent of frequency and pressure which is very difficult to obtain in practice. In both the apparatus developed by King and Churcher (J.9) the moving coil type with permanent magnet is used. The velocity of the moving coil $V_{mt} = \frac{P_a}{M_t W}$

where P_a is the maximum value of the alternating air pressure and M_t is the mass of the moving sgstem. The maximum voltage generated by the mass controlled microphone is

$$\frac{P_a \; N \; B}{fmt} \quad 10^{-8} \quad \text{volts (max)}$$

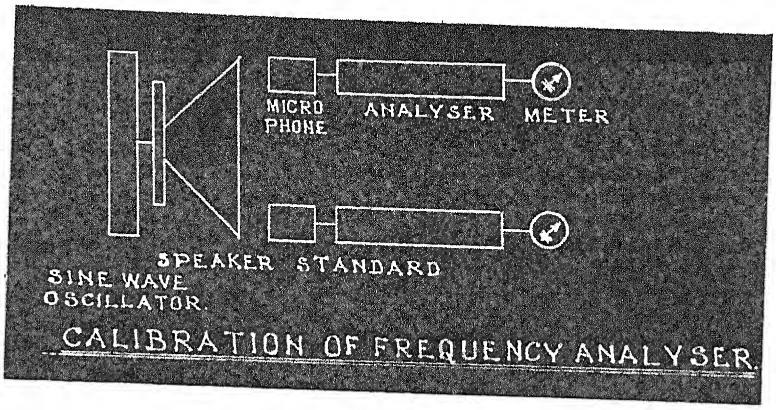
where N = turns on moving coil.

r = radius of the moving coil and B the average flux density in the air gap. In the mass controlled microphone the use of the large mass to extend the useful range of the microphone to very low frequencies has the effect of reducing the sensitivity at all frequencies. Secondly in the measurement of noise containing small high frequency sound and large frequendy ramble, the latter is augmented by the high sensitivity of the microphone, while the small high frequency sound, reduced by the low sensitivity of the microphone give amplitudes too small to measure. To correct this the mi amplifier can be arranged to give an amplification proportional to fréquency by using a suitably proportional mutual inductance to couple the first two values of the amplifier. Even if the variation of the sensitiveness of micorphone is not proportional to frequency, with some difficulty it is possible to make the microphone amplifier unit to work satisfactorily. If the output of microphone were to fall beyond a certain frequency the amplifier may be present to give higher amplification for those frequencies.

In order to be able to make measurements of sound intensity it is necessary that each link of the chain from the sound energy to the deflection of the find Galvanometer should be of known permanent performance. The units involved in this transformation are the microphnne, the amplifier, and the frequency analyser.

At the first instance we must know the voltage generated by the microphone when acted upon by a sound of

unit intensity at any frequency over the range required for tests. Then the frequency response of the amplifier and also an overall calibration of the whole apparatus in meter deflection per unit sound input must be obtained.



CALIBRATION OF FREQUENCY ANALYSER.

The Rayleigh disc experiment is the absolute method of studying the characteristic of the microphone. In this the principle of standing waves of sound is adopted. The Rayleigh disc made of mica is suspended by a fine phosphorbronze suspension from a torsion head. The disc is hanging in a tube about 1½ meters long and about 20 cms in diameter. The disc is suspended in a tube where the velocity is a maximum. Since Rayleigh disc does measure particle velocities as deduced by Koning's formula provided that

(1) the disc is then compared to the diameter.
(2) the ratio of the thickness to diameter of the disc is of the order of 0.01.
(3) The diameter of the disc is small compared with the wave length of the note under test.

(4) The disc diameter is small compared with the diameter of enclosing tube; a ratio of diameters of three being satisfactory.

The conditions for standing waves in the tube are satisfied if there is any whole number of half wave lengths between the fixed reflecting ends. The microphone and loud speaker are mounted on 2 discs of hardwood about 4 cms thick. The cone of the microphone was filled in by a flat celluloid diaphragm supported from the cone by ribs of same mabrials to increase the efficiency of the system. The joints between the tubes and wooden discs were filled with plasticine. By noting the response of the amplifier and frequency analyser with a sound wave of amplitude recorded by the deflection of the Rayleigh disc the sensitivity of the microphone was determined in millivolts per dyne per sq. cms.

In the same setting of the apparatus the 3rd, 5th, 7th etc. harmonies of the fundamental adjustment can be made. Even harmonies cannot be adopted since the disc must be at a velocity loop. The disc diameter, tube diameter and lenghth of the tube will have to be changed while using higher frequencies, so as to give better results. The power output and frequency of oscillator must be continuously variable and at the same time remain stable at any setting. The microphone dimesnions can introduce error in the calibration and so correction factor must be applied.

After calibrating the microphone it is to be connected

to the amplifier. Twisted leads in flexible metallic tubing like lead sheathing prevents inductivé and capacitive couplings. In view of the complex sounds to be analysed no definite slectivity requirments can be stated for the circuits of the analyser. But higher selectivity is better.

The room where the machine and testing apparatus are located must be free from extraneous noise and reflections. If the reflections are not reduced the incident and reflected waves travelling with equal and opposite velocities produce an interference pattern. The pressure (vs) distance from source from source curvé will be a wave superimposed on a rectangular hyperbola. If refelection is suspected it is possible to trace out a mean curve of the wavy curve and thus reduce the effect of reflection. This will havé to be done for more of the tests on machine where a number of frequencies are present. Although the walls of a building are highly reflecting the interference trouble may not be prohibitive, if the building is large and the machine is not close to walls. A little considerations will make it clear that the sound intensity measured near a source is dependent on the presence of the other bodies in the sound field which would cause reflection. Even an open space does not always prove to be effective due to the presence of wind. Hence such measurements must necessarily be carried out in accoustically treated rooms with the sound source kept far from the side walls. Directional microphones do sometimes prove to be more effective.

Hence when experiment is conducted microphone is kept at a standard distance from the machine. Two positions of the microphone are along the axis of the machine and the other perpendicular to this line. Readings from opposite positions will almost be identical. Since the sounds are peculiar to rotation of machine each frequency is divided by R.P.S.of rotor. This is very helpful in tracking the component sounds to their respective sources (J.8)&(J.9).

7.6 Methods of Elimination.

Different methods have been adopted to reduce the noise emitted by a machine but in most of the cases it is seen that materials are not fully utilised when noiseless operation is aimed at i.e noiseless machine will have to be larger in size. The different methods are evolved, from the discussion given in the previous sections.

Oscillations of laminations of few thousands of a m-m can produce acute sound if the lamintions are not properly clamped. When the stator is mounted on the ribs of the stator casing the vibrations in Stator are transmitted through ribs to the then housing which is a good sound radiation. Hence slots should be selected to avoid radial forces. Even if the stampings were to sit tight on casing without using ribs, radial forces, if present will cause noise. Also even if the rotor is rigid this will also cause noise due to play in bearings. Also it is to be noted that same lamination is to be utilized by manufacturer for same size motor even with different poles. But investigation

shows that slot combinations can be found which can be used for different number of poles upto a certain size. Exact centering and balancing is essential for quite operation. Journal type bearings have been found to give better results than roller type. To avoid transmission of vibration through the machine supports special mountings have been used providing torsional elasticity with a maximum of radial stiffness. Spring supports and rubber mountings are very commonly used for this purpose.

Regarding the improvements on slots, machines with close slots are found to be quieter in operation and Brown Boveri uses closed slots exclusively for noiseless squirrel cage motors. Increasing airgap reduces the higher slot oscillation and this method of noise reduction on Induction Motor can be done only at the expense of power factor. Also high saturation cannot usually be allowed on noiseless machines. Further under certain conditions a motor having a slot number which is theretically undesirable mayrun practically noiseless, while if same slot number is used for another motor would lead to noise. A slot combination theoretically suitable for a given type may prove equally good for another type of motor. The shape of the slot also affects the noise level of the machine and some manufacturers have attempted improving on the slot-shape based on the principle of flux distribution. Good winding distribution as already stated is a very efficient and easy method for suppressing certain harmonic frequencies. Skewing the rotor bars is

common among induction motor and this method is effective in reducing the noise level apart from the other advantages. To reduce deflection and vibration, from early days the shaft cross section were increased together with increased depth of stator core. But the present day trend is to make the motor light and still succeed in manufacturing a quiet motor.

Gathering conclusion about proper slot combination it can be seen that (J.3)

(1) when the slots differ by one or by the number of poles ±1 transverse vibrations may occur.

(2) when slots differ by half the number of poles, torsional vibration and noise may occur.

(3) When slots differ by half the number of poles torsional vibration and noise may occur also. rumbling noise accompanied by critical vibrations.

(4) The chance that noise occurs in the working range is greater than with small number of pobes, high speed, and with small transverse or torsional critical speeds. When the noises occur the motor is practically useless.

(5) Any rotor with the slot numbers divisible by number of pole pairs offundamental and which differ from the number of stator slots by more than the number of poles will probably be a quiet motor. (J. 3).

CHAPTER VIII

Power Losses in Induction Machines.

8.1 Introduction.

In the study of the efficiency of electrical machines the knowledge of the sources of power loss within the machine is imperative. Some of the losses, however, are quite complex and do not readily yield to a mathematical treatment or to an experimental determination. This is particularly true in regard to some forms of load loss in induction motors. All losses in induction machines, except friction, windage, and copper resistance loss, are intimately connected with the magnetic flux within the machine. It is obvious, therefore that a detailed study of flux distribution is a logical way of approach to the study of the problem of losses. caused by the changing flux, all losses can be conveniently classified into two groups; losses occuring at the fundamental frequency, and losses occuring at high or tooth frequency. Both of these groups involve a certain amount of iron losses (hysteresis and eddy current), and copper eddy-current losses. (J.2).

8.2. Fundamental Frequency Losses.

In the fundamental frequency losses there are included losses caused by the uniformly rotating main flux, and

leakage fluxes in both rotor and stator, varying at the fundamental frequency. These losses occur at line frequency in the stator and at slip frequency in the rotor. In any point of the magnetic circuit the flux may be considered as consisting of 2 components: a radial components: a radial component and a tangential component: (J. 11) The radial component in the teeth and slots constitutes the greatest percentage of the total flux in the region of the maximum flux density. This component is active in producing all useful currents and voltages in the Induction motor windings. The tangential component constitutes the greatest percentage of the total flux in the region of a minimum air gap flux. Because of the presence of this tangential component the flux in the teeth and slots never dies down to zero value while reversing in direction, but varies elliptically (J. 8).

In any volume of a conducting or magnetic (or both) material, losses of energy will necessarily be caused by any change in the pattern of the magnetic flux penetrating the volume, provided the hysterésis and resistance effects in the material are not zero. The reverse statement is also true, i.e, no loss can be caused in an insulated volume of any material by moving it in a magnetic field at a uniform speed provided the flux pattern in the volume remains the same. The truth of these statements

regarding a magnetic material can be readily seen if we remember that a hysteresis loss is caused only by a change in the magnetic state of the material, the chnage being either in intensity or the direction of the field. In a conducting material, the change in flux pattern causes different electric potentials to be generated in different parts of the volume. These potentials produce eddy current losses within the volume. It will be seen that a moving flux of a constant space distribution is not capable of producing eddy currents within an insulated volume, because the rate of cutting the flux is the same in this case in all parts of the volume.

In the light of the foregoing statements it can be seen that both eddy current losses and hysteresis losses are produced in the machine iron and eddy current losses in the copper conductors, because the flux pattern changes at the fundamental frequency; as the flux rotates. In regard to eddy currents in the conductors, it may be convenient to consider separately two mutually perpendicular components of flux; radial and tangential. The cyclic variation in the radial component produces opposite potentials at the sides of the conductors in the slots. Thus the eddy current loss that is due to this mode of flux variation may be reduced by laminating the conductors in

the radial direction. In a similar manner, the variation in the tangential component produces opposite potentials at the top and bottom of each conductor. The remedy is to lamiaate the conductors in the tangential direction.

The losses described above occur at no load as well as at load. When the motor is loaded an additional fundametal frequency loss takes place. (J. 9) This loss is caused by leakage fluxes. Surrounding each individual slot, this flux crosses at the top of each slot in the tangential direction. While the current in the conductors reduces from amximum to zero value and the field disappears, the lower (internal) part of the slot is cut by a greater flux than the upper (outer) part, the difference between the two fluxes being equal to the flux erossing the slot when the field is a maximum. When no saturation in the iron is present, the losses caused by this leakage flux are proportional to the square of the load current, and therefore, their effect is to increase the effective resisitance of the circuit by a constant amount. With the stator or Rotor slots closed by a thin iron bridge, the saturation in the latter increases the density of flux within the slot (J.9) This causes additional losses which appear with load, and which cannot be accounted for by adding a constant amount to the effective resistance of the circuit. With bar wound stators theese losses may reach a considerable value.

In low slip rotors, the fundamental frequency losses hardly amount to any appreciable value. (disregarding the copper loss that is due to the main useful current.), because of the low frequency of the loss producing fluxes. However, it must be pointed out that the leakage fluxes, being affected by saturation in the iron, must have higher harmonies in them.

8.3 High Frequency Losses.

Three modes of tooth frequency flux pulsation may be indicated which produce a certain amount of iron loss. They are:

1. Surface, or tooth tip pulsation.
2. Tooth flux pulsations (flux pulsations penetrating the whole length of a tooth)
3. Pulsations of flux in the core.

All these modes of flux pulsation constitute a variation of flux through a small cycle superimposed on the fundamental flux wave. As the cause of the flux pulsation is a pulsatio n of the permeance of the magnetic path, the amplitutde of the high frequency pulsation depends upon the magnitude of the fundamental flux at the instant, i.e it varies at the fundamental frequency. Considering the hysteresis pulsation loss alone, it must be pointed out that the hysteresis loop of the pulsating flux is, in general, unsymmetrical due to the p of the fundamental flux. Further more, with metrical hysteresis loop of the losses are re

greater than with an ordinary symmetrical loop of the same flux amplitude (J. 10) The eddy current loss caused by the high frequency flux variation shows also an effect unobserved under ordinary frequency conditions. Because of the skin effect in the iron, the eddy current losses increases less rapidly than the square of the frequency, for frequencies above 100 cycles. This influence of the skin effect relatively decreases the eddy current loss at the tooth frequency. It must be noted that, on the whole, the x unsymmetrical hysteresis and the skin effect tend to equalise each other (J.4).

a. Surface flux Pulsations.

Of the 3 manners of flux variations mentioned above, the first one is quite similar to the pole face flux variation in salient pole machines that is due to passing teeth. It does not penetrate the whole length of the tooth, but disappears at a certain distance from the tooth tips. The magnitude of the surface flux pulsations depends upon the fringing of the flux in the air gap, i.e upon the geometry of the magnetic path. Thus, the surface flux pulsations in the rotor increase with an increase in the width of the rotor tooth tip, but decrease with an increase in the air gap, because of the effect of flux fringing. At the same time a relatively narrower slot in the stator causes a small magnitude of surface flux pulsation in the rotor. With both Stator and Rotor slots closed, the

flux pulsation is reduced to a minimum. With only the rotor slots closed (or semi closed, as is commonly the practice) the surface flux pulsations in the stator are practically negligible; but may amount to a considerable value in the rotor (J. 4).

b. Tooth flux Pulsations.

If an individual tooth is watched, while the teeth of the opposite magnetic member are passing by, it will be noticed that the permeance of the magnetic path terminating in the tooth in question is oscillating through a definite cycle which repeats itself with every passing tooth.

c. Pulsation of Flux in the Core.

The arrangement of teeth in the stator and rotor may be such that the motion of the rotor causes some tooth frequency variation in the total permeance of the magnetic circuit. With the ratio of rotor teeth to stator teeth markedly different from unity this variation, if any, is small, but when present it may cause the total Mutual Flux to pulsate. Actually, the pulsations in the total permeance of the magnetic circuit are more likely to manifest themselves as high frequency ripples in the magnetising current than as an actual variation of the total flux. In either case, however, some loss is caused by the pulsation of the total permeance of the magnetic circuit. If this loss appears as the I^2R loss that is due to high frequency component in the exciting

current the energy of the loss may be dissipated as heat, not necessarily within the motor, but in the line and the supply generator conductors as well. But no matter where it be consumed, within the motor it will appear as a mechanical load on the motor, and therefore it will always form a part of the input of the motor. (J.7).

8.4 Copper eddy currents and their effect upon the flux pulsations.

It has been shown experimentally (J.8) that the effect of the eddy currents is to reduce the amplitude of the flux pulsations and at the same time to increase the power loss. Three kinds of high frequency flux pulsations which produce eddy currents in the copper conductors will now be considered in turn:

1. Pulsation of the slot leakage flux.
2. Pulsation of the tooth flux proper.
3. Pulsation of the fundamental leakage flux.

8.5 Effect of Slot leakage flux Pulsations.

Slot leakage flux can be considered as consisting of radial and tangential components. The amount of this radial component in the slot pattern obviously depends upon the position of the stator tooth relative to the rotor slot. When the stator tooth is directly opposite the rotor opening, the thin rotor tooth shoulders may become saturated easily; and a considerable part of the flux may penetrate

the slot. Thus, at the relative position of the Stator and rotor teeth the amount of the radial component of the tooth leakage flux is a maximum. When the stator and rotor teeth are in a position directly opposite each other, most of the flux passes through the teeth and only a small part penetrates the slot. Thus, at the position of the rotor, and stator teeth, the radial component of the slot leakage fluxes is a minimum. (J.7).

The tangential component of the slot leakage flux is generated in excatly the same manner as the tangential component of the fundamental frequency leakage flux. Whenever a flux density in any 2 adjacent teeth is different the tips of the teeth are at different magnetic potentials, which causes some of the flux to cross the slot from one tooth to another. As the sign of the difference in flux density of any 2 neighbouring rotor teeth varies twice during the time one rotor tooth moves one stator slot pitch, the direction of the tangential component in the slot between the teeth varies according twice during that time. The pulsation of the tangential slot leakage flux, togethr with the pulsation of the radial component, cause a part of the high frequency eddy currents.

8.6 Effect of the Tooth Flux Pulsations.

In a squirrel cage rotor the flux pulsation in the teeth is one of the causes of a circulating current in the short circuited bars. Every 2 neighbouring bars represent a closed low resistance electric circuit, which links with the magnetic flux in the tooth between the bars. High

frequency pulsations of flux in the tooth induce high frequency pulsations in the bars, whichin turn cause circulating currents. The effect of these currents is to set up fluxes of such a magnitude and direction as to oppose the flux variations producing the currents. The remaining flux pulsations in the teeth are just sufficient to induce voltage to overcome the resistance and the inductance of the bars. Since both the resistance and leakage reactance of the bars are extremely low, the amplitude of the residual flux pulsations is very small (J.8). Thus at least as far as the squirrel cage rotor is concerned, the tooth frequency loss takes place not in the form of iron loss, but mostly in the form of copper resistance loss in the bar windings, due to eddy and circulating currents (J.7).

In a wound rotor, especially if the rotor conductors are comparatively small, the damping effect of the tooth frequency currents is not so pronounced as in a squirrel cage rotor, and the tooth frequency iron loss is probably an appreciable item.

8.7. Pulsation of the main leakage flux.

One more possible kind of high frequency eddy current loss is that due to the pulsation of the main leakage flux. It has been previously stated that the main leakage flux occurs principally in the form of a flux surrounding each individual slot, crossing the slot at the opening. The leakage flux that coosses the air gap is commonly known as

as a Zig Zag leakage flux.

The reluctance of the path of the leakage flux which crosses the air gap depends upon the relative position of the stator and rotor teeth. When the teeth of one magnetic member are opposite slots of the other, the reluctance is a minimum. When the slots and teeth of the stator oppose the slots and teeth of rotor, respectively the reluctance is a maximum. This partial variation of the reluctance may cause some pulsation in the magnitude of the leakage flux, and of the pattern of the flux in the slots. As the amount of leakage flux depends upon the load current in the conductors, the possible leakage flux pulsation loss should vary approximately as the square of the load current.

8.8. <u>Tooth frequency Losses in the Stator</u>.

Flux pulsations in the Stator present essentially the same features, as that of the rotor. The damping of the rotor flux pulsations by the tooth frequency rotor current increases markedly the pulsation of flux in the Stator (J.1). If there is any decrease in the rotor pulsation loss that is due to the damping currents this decrease is made up with excess by the increase in the stator tooth frequency loss (J.6.).

8.9. <u>Losses due to minor fields</u>.

There are some minor losses in induction motors which have not been described above. Only 2 of them

will be briefly mentioned here. One of these losses occurs in the end laminae of both rotor and stator, and is caused by the end fringing of the main flux. An electromotive force induced by this fringing flux causes currents to flow in the plane of the end laminae because, as the field changes its strength the lines of fringing flux cut the iron in a direction perpendicular to the planes of the laminae. This loss occurs at the fundamental frequency.

The other loss occurs in squirrel cage motors only, and is due to the nonsinusoidal flux distribution in the airgap. Ordinarily, the stator winding is placed in such a manner that the flux distribution only approaches a Sine wave form (e.g it may have a trapezoidal shape). It may be shown that in this case the total air gap flux has irregular, highe frequency components. When the air gap flux apparently moves at synchronous speed of the machine, only the true sine wave component participates in the motion, and is useful in producing the driving torque of the motor. The effect of the irregularities in flux distribution is to produce circulating currents in the squirrel cage bars, which increase the no load loss and make the flux wave more nearly sinusoidal.

8.10 Stray Load Losses.

If the external causes of loss in induction motors are excluded, such as those caused by an unbalanced voltage, nonsinusoidal Voltage wave, etc., the losses described above seem to cover all possible modes of appreciable and nonproductive expenditure of energy within a machine. It seems

to be logical, therefore, to look for the stray load losses, i.e. the additional losses appearing with load, among those previously described. In order to do this, it is necessaryto see which of all the losses vary with the load, and in what manner. (J.3.)

First, it will be noted that some of the losses actually decrease with load. Consider, first instance, the manner in which the fundamental frequency flux changes with load. With a constant impressed voltage, the flux which links with the primary winding decreases slightly with load, because of the IR drop in the primary winding. This causes a slight decrease in the fundamental frequency iron loss.

On the other hand, there are losses which definitely increase with the load. As the leakage flux is the only loss affecting factor that changes with load, it must directly or indirectly account for the increase in the losses with load, i.e., it must account for the stray load loss. In the first place, the leakage flux produces a fundamental frequency eddy current loss in the rotor and stator conductors. With a bar wound stator, this eddy current copper loss may reach an appreciable value. In rotor bars this loss hardly amounts to much, because of the low rotor frequency. In any case, the effect of this loss is to increase the effective copper resistance by a constant amount.

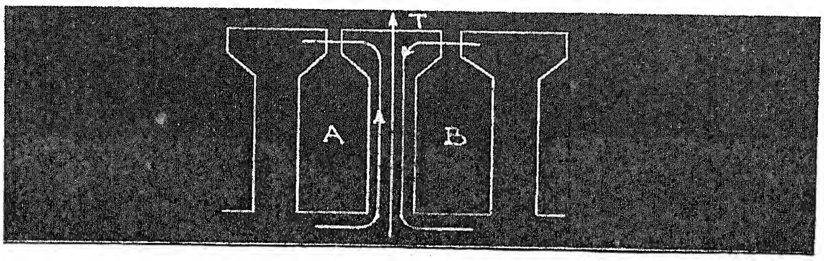
Furthermore, the leakage fluxes are likely to produce a high value of saturation in the tooth tips and, particularly, the thin tooth tip shoulders of semiclosed or closed

slots. This saturation in the tooth shoulders causes a redistribution of flux in the airgap, and, what is more important, in the slots, every time the leakage flux exceeds a certain value. This occurs every half cycle. Accordingly, the load leakage flux in the slots, when iron tooth tip shoulders get saturated, has pronounced higher harmonics in it. This increases the eddycurrent losses in the copper conductors that are due to the fundamental leakage flux, and may account for the fact, that machines which have closed slots in both rotor and Stator show a high value of the stray load loss. (J.10).

8.11. Other contributing factors.

Another phenomenon that may contribute to the stray load loss is the tooth pulsation of the main leakage flux. Consider the ~~extremey~~ case of a closed slot rotor. At low load, when the rotor leakage flux is low, the iron bridges that close the slots are not likely to become saturated. Such being the case, the iron bridges represent a sufficient magnetic shielding against any effect of the stator teeth upon the distribution of the leakage flux in the rotor slots; i.e no tooth pulsation of the rotor leakage flux is possible. When the motor is loaded the rotor leakage flux assumes an appreciable value, the thin iron bridges becomes saturated easily. Tooth pulsation appear in the leakage flux that crosses the slots. The eddy current power loss caused by these pulsations contributes to the stray load loss.

Within the stator and rotor teeth themselves, leakage flux may cause some distortion in the main flux. The figure is intended to illustrate this point. Though the leakage flux tht is due to the current in A is distributed through the cross section of tooth T, its density is not uniform throughout the cross section of the tooth, being greater near A and less near B. Similarly, the leakage flux density that is due to B is greater near B and less near A. If the direction of the main flux is as shown by the arrow, it is obvious that the resultant flux is of greater density near A, and of asmaller density near B. Thus the effect of leakage fluxes within the teeth is to distort the resultant tooth flux, crowding it to one side of the teeth. This effect increases with load and may account for a part of the stray load losses.



As may be seen, the stray load loss in induction motors, whose satisfactory method of experimental determination is still a problem, is very likely a composite effect of several factors. The relative importance of these factors can hardly be discussed wihtout further experimental investigation.

CHAPTER IX.

Temperaturé Rise.

9.1. Introduction.

The maximum allowable temperature rise of any electrical apparatus dictate its overload capacity. (B.1). Any further overload for a longer time will result in the destruction of the insulation thereby causing the breakdown of the machine. The modern theory on which the thermal rating of the dynamo electric machinery is based considers that there is a certain working temperature, above which the insulating materials undergoes thermolysis. This is the critical temperaturé for the type of insulation employed. Below the critical temperature, it is supposed that the working life time of the insulation is indefinitely long. Motor insulations are defined in three major classes. A, B and H for which limiting hot spot temperatures of 105°c, 130°C and 180° C in continuous service are recognised. The other reasons for limiting the temperature rise of a motor are; (B.2) the resistivity of copper increase linearly with temperature, and the different thermal expansions of iron, copper and other materials give rise to mechanical stressés and displacements, that cause progressive deterioration. The resistance of copper increases in the ratios of 1 to 1.24 to 1.33 to 1.61, as the temperature is increased from 40 to 105, to 130 to 180°C

$$R_T = R_{75} \frac{235 + T}{310} \quad \text{where T is } ^\circ C$$

Thus, a motor designed for the limiting temperature of

class H insulation is handicapped by 22 percent greater copper resistivity than one similarly designed for class A insulation.

American standards assume a normal ambient temperature of not over 40°C and allow a temperature rise at full load of 50°C by resistance method. (40°C by thermometer) for a general purpose motor, 60°C rise of resistance is allowed for special purpose motors, with class A x insulation 80°C from Class B, and 120°C for class H insulation. The values allow for diffrentials of 5°C ø 10° C and 20°C respectively, between the hot spot and the (average) temperature measured by the resistance method.

<u>Methods of temperature Measurements</u>.

Three methods of measuring temperatures are recognised by British standard Specification 169/1925 and they are (B. 3)

Thermometer Method

Resistance Method

Embedded Temperature Detector Method

<u>Thermometer Method.</u>

When the thermometer method is used the temperature of the machine shall be measured by thermometer applied to the hottest accessible surface of the stationary parts of the machine during the test period, and by other thermometers applied to the accessible surfaces of the rotating parts as soon as the machine is stopped, after the test. The thermometers may be Mercury or alchol bulb thermometer. (B J.9).

When the bulb thermometers are employed in placed where there is any varying or moving magnetic field, alcohol thermometers should be used in preference to mercury thermometers as the latter are unreliable under these conditions.

In all cases the bulb of the thermometer, except at the point of contact shall be covered with a pad of felt, cotton, wool or other non conducting material 1/8" thick extending at least three quarters of an inch in every other direction from the bulb and pressed into contact with the surface to which this is applied, to prevent loss of heat by radiation from the bulb (J.4).

Resistance Method.

In this method the mean temperature rise of the winding shall be determined by the increase in the resistance of the windings themselves, and checked by thermometers applied to the accessible surface of the windings to ascertain whether there is any higher local temperature. The highest of the temperatures thus found shall be taken as the temperature by resistance method (J.4)

The temperature of the windings as measured by thermometer before commencing the test should not differ from that of the cooling medium. The initial resistance and initial temperature of the winding shall be measured at the same time (J.4).

N.B.:- Since the resistance of copper over the range referred to in this specification varies in direct proportions to the temperature (above - 234.5 ^{o}C) the ratio

of hot to cold temperatures may be obtained from the ratio of the resistances by the formula:

$$\frac{R_2}{R_1} = \frac{T_2\ C \ - 234.5}{T_1\ C \ + 234.5} \quad \text{where}$$

R_2 = Resistance of wi lings hot

R1 = Resistance of windings cold

T2 = Temperature of windings hot

T1 = Temperature of windings cold.

Embedded Temperature Detector Method.

Embedded temperature Detectors are resistance thermometers or thermocouples built into the machine during construction at points which are inaccessible when the machine is completed. This term does not include the necessary measuring instruments.

When the internal temperature of a machine is to be measured by Embedded Temperature Detectors, at least six detectors should be built into the machine, suitably distributed around the circumference within the slots, all reasonable efforts consistent with safety being made to place the Embedded Temperature Detectors at the various points at which the highest temperatures are likely to occur.

When the winding has more than one coil per slot; the detector is placed between the upper and lower coils in the slot.

In the case of a winding having only one coil per slot, the detector is placed between the outside of the coil and inside for the slot lining at the bottom of the slot.

The temperature of a machine shall whenver possible be taken during working as well as after stopping the machine; the highest temperature thus obtained shall be adopted. When successive measurements show increasing temperatures after shut down, the highest value shall be taken.

In the case of rotating parts, if the intergal between the instant of cutting off the power and measuring the temperature after the machine comes to rest, suitable corrections shall be applied so as to obtain as nearly as practicable the temperature at the instant of shut down. (J. 2).

Precautions to be observed.

When the temperature rise of a machine is measured by the increase in resistance as a commercial test it is to be observed that more than commercial accuracy is required, if accurate deddiction of temperature are to be obtained. (J.4)

The temperature coefficient of increase of resistence of copper for $1^{o}C$ rise is approximately 0.4% so that instruments must read within tthis percentage if the temperature is to be deduced to within 1^{o} C. The determination of resistance therefore to the third significant figure does not always determine the temperature to within $1^{o}C$. For instance, if the resistance of a coil is about 1 ohm its resistance at a higher temperature im order to determine that temperature with accuracy must be measurable

down to four thousandth of an ohm. (J.6)

In order to determine the temperatures of the windings when hot, an accurate measurement of resistance and associated temperature must be taken when the windings are cold. Special precautions should be taken in the case of large machines in view of the inaccessibility of the windings, and the possibility of unequal temperatures in different parts of the machine. These cold readings therefore should be taken after the machine has been standing for sometime, so that it may have assumed the temperatures of the surrounding atmosphere. In the case of large machines, this length of time may quite well be 24 hours or longer, and in engine rooms which are subject to fluctuation of temperature, further precautions will be necessary.

In the case of rotor windings the resistance of the brush contact is appreciable and should be eliminated from any measurement of the resistance of the windings. In order to do this a special brush should be used which is insulated from the rest of the brush gear and serves only to carry the small voltmeter current from the ring surface.

9.4 Advantages of the Temperature Measurement By the Thermo Couple Method.

In specifying the full load temperature rise of the winding of a single phase and polyphase integral horse power motors. NEMA publication MG 1-1949 maintains a temperature difference between rise bythe thermometer and rise by resistance of $10^{o}C$ except in the case of totally enclosed

fan cooled motors where this difference is 5°C. A possible reason for this small difference in temperature rise by resistance and that by thermometer of totally enclosed fan cooled machines might be the fact that these motors are usually single end ventilated and the windings on the fan end are much cooler than on the other end.

During the last 10 years manufacturers have produced general purpose open type single end ventilated motors and the question arises as to whether the 10° C or 5° C temperature differential should be adopted for the open type motors of single end ventilators. When the original AIEE standards were adopted most motor manufacturers were using glass tube thermometers for measuring the temperature rise of the motor windings, while today they are using surface thermocouples for economy and convenience.

As per the definition of "Thermometer Method of Temperature Determination", it should make no difference whether the temperature is measured by the glass tube thermometer or by the thermocouple. In practice however the definition is not followed and the thermocouples are placed on the hottest spot of the machine accessible to the thermocouples, thus obtaining much higher temperature nearer to the actual temperature of the winding.

E.R. Summar in his Paper, (J.4) " Determination of Temperature rise of Induction Motors" submitted the results of temperature rise tests on 228 motors from 10 H.P Those tests show that the temperature rise by thermocouples

show as much or more than the rise by resistance.

Veinott come to the following conslusion drawn from the results of 585 temperature tests taken in fractional horse powdr Motors.

The temperature rise of single phase Induction motors measured by surface thermocouples average as much or more than the rise by resistance.

The surface thermocouple method of temperature measurement should be recognised as a distinct method instead of classifying it with the thermometer as done in the present AIEE standard No. 1. For purposes of rating the limits assigned to this method of determining the temperature rise may be the same as those assigned to rise by resistance.

If the method of determining the temperature rise of single phase motors is to be changed, the present limit of 40°C by thermometer could be changed to 50°C by resistance on 50 C by surface thermocouple without appreciably affecting the actual temperature rises.

Out of the 92 commercial temperature tests done by G.P. Potter on 4 pole 60 cycle single end ventilated open type motors it was found that 50% of the tests show the difference between the temperature rise by resistance and the temperature rise by the surface thermocouples was 2°C or less and in 74% of tests this difference was 3°C or less (i.e) for all practical purposes the temperature rise by resistance is the same as that by surface thermocouple on this type of motors (J.1)

The temperature differentials in single end ventilated open type motors are substantially the semd as far double end ventilated machines and no changes are desirable in the temperature standards on that account.

The temperature rise measurement by surface thermocouples which are used commonly for this purpose, gives much more accurate results and is more convenient and dependable. So G.P. Potter recommends (J.3) to abolish the " Thermometer Method of Temperature Determination." for rotating electric machines and to define " The thermocouple method of Temperature Determination" and specified in the place of "Thermometer Method of Temperature Determination", using the same present temperature limits, now specified for the temperature rise by resistance method.

Ventilation.

Heating is frequently the factor which determines the size of electrical machines and structures wholly unnecessary mechanically are forced on the designer by temperature limitation. As improvements were made in the characteristics the limit of temperature rise becams more important. For the reduction in sizes of machines insulation capable of standing higher temperature must be used and methods of cooling may be devised to dissipate larger losses from the same active material. Unrestricted path must be provided not only for electric flux and magnetic flux but also for thermic flux.

Various types of enclosures are commonly used.

The open type is adopted where skilled attendants are handling the machine. The screen protected type has the various openings covered with expanded metal and is very common amongst small and medium sized machines. The drip proof type of enclosure is intended to protect a machine against falling water or particles of dirt as in boiler houses and similar positions. Pipe ventilation used for industrial motors are employed to obtain a supply of clean air as a protection against chemical fumes and dust. Totally enclosed machines with closed air circuit gives quite good ventilation.

As regards ventilation the radial, the axial and mixed flow are adopted (J.7) Attachment of blades to the rotors of squirrel Cage Induction Motors is an example. In this system air is drawn in over the bearing housings, passed through and around the stator end windings and expelled through openings in the yoke. For core lenghts longer than 8" this arrangement is not satisfactory. Radial ducts are used to overcome the difficulty of hot spots. For smooth bore stators and rotors the radial duct scheme has a number of disadvantages. It requires close attention during manufacture to ensure that the correct number of laminations are placed in each packet so that the ducts come opposite to one another; when the rotor is assembled in the Stator. Also at peripheral speeds over 8000 ft. per minute the passage of air from the rotating to the satisfactory duct tends to make

the machine noisy. Although the air entering the stationary duct has a velocity which renders it very effective for absorbing heat its small volume causes it to reach the temperature of the duct after which it can pick up no more heat. Under such circumstances a separate fan is also used. Purely axial ventialtion with axial ducts in laminations of both stator and rotor are also used for Induction Motors. If a fan is mounted at one end of the machine this system gives a number of advantages over the more common forms of radial ventilation. The mixed type will have the disadvantages of both but is better suited when there is a good amount of iron behind the slot. Also fan should be so positioned to avoid opposing effect of the fan head and the natural head of the ducts.

As mentioned the different methods of determining the temperature of the machine are thermocouples imbedded in coils, thermometers and resistance variation and all of them indicate different values. Hence, for fixing the standard if method of thermocouples are employed the basis will be hot spot temperature and if thermometers are employed the basis will be mean temperature. Also various types of coils have different ratios between their hot spot temperatures and average termperature. The hot spot temperature is made up of temperature difference between the hot spot and the dissipating surfaces over which the air flows, temperature difference between dissipating surface

over which the air flows temperature difference between dissipating surface and cooling air.

Temperature rise of cooling air.

In a axially ventialted machine air passing through the rotor collects heat from the stator inlet end winding and rotor ducts while air passing through the stator collects heat from the stator core and the outlet winding. Heat flow from the interior occur axially, radially and circumferentially. When the temperature of a machine is higher in the embedded portion than in the end winding axial heat flow results. Also in machines with radial ducts heat flows across the lamina tions towards the surface of the ducts. The temperature of gradient across laminations vary with the amount of insulation between laminations. Radial heat flow occurs in axially ventilated machines. It also occurs along armature and stator teeth due to the actual iron loss generated in the teeth and also due to copper loss in adjacent slots. In the case of armatures mounted on spiders some heat is transmitted down the spider arms and collected from the surface of the Spider by the cooling air. Although spiders usually have alarge dissipating surface, their efficiency is spoiled by the restriction of thermic flux to a small cross-section and thus causing excessive temperature gradient. In some cases the dissipating surface in contact with air are the outer surface of the laminations and inner surface of the yoke. As all heat flowing into the yoke must pass

through the 4 narrow arms on which the laminations are mounted the thermic flux is again restricted. When the rotor or stator is incapabble of dissipating all the heat some transference takes place across the gap. Circumferential heat flow occurs in both radially and axially ventialted machines from the conductors in a slot to the coil wall. The temperature gradient with round wire type coil can be improved by filling up air spaces with bitumen. When coils narrower than width of slot are used temperature gradients may exist on account of the air spaces.

The cooling media and the cooling surfaces are then to be studied in connection with heat dissipation. The cooling media most frequently mused is air. It may be drawn through machines by natural windage or by fans and after picking up the heat it is usually carridd a short distance away by its momentum. Hydrogen is also used in place of air in closed circuit due to its higher thermal conductivity and lower windagé losses. Air and water in series with one another is also used. In this case heat is picked up by the air in the machines transferrdd to the water in the cooler. If the temperature difference between the hot spot and the dissipating surface is a considerable proportion of the total allowable temperature rise of the hot spot, the quantity of the cooling medium will have to be larger than would normally be supplied. In pipe ventilated machines the air temperature rise may

reach 65% of the allowable temperature rise of the apparatus and 30 to 50% is more usually adopted. But temperature rise of the cooling medium can never be allowed to approach the allowable temperature rise of the machine. Thus for 40 C temperature rise 1 gallon of water per minute for purely water circulated machines and 70 to 100 c.ft of air per minute for purely air cooled machines will be necessary. Doubling the air quantity cannot halve the temperature rise and in fact if quantity of air is increased by 20% the drop in temperature will be between 4% and 12%. T hough turbulences increases heat dissipation it is unsatisfactory to rely upon very high rate of dissipation at certain localised points. Both water gauge pressure due to turbulence and air volume are necessary heat transfer at higher air speed and turbulence is due to the reduction of the thin stationary air film adjacent to the duct walls in which most of the temperature difference occurs. Next considering, the cooling surfaces, ducts with smooth surface have lower coefficient of dissipation. The air passage is through the belt below the slots.
2) the portion between the teeth.

The dissipating area of the duct below the slot includes both walls of the duct both sides of the spaces and the portion of the coil corresponding to the bottom of the slot. For the portion of the duct between the teeth, both walls of the duct, spaces and sides of the coils adjacent to the duct should be considered. Radial

ducts in machines with smooth bore stators and rotors as in induction motor are very ineffective regarding cooling. In machines with air gaps of less than 1/8" the flow of the air through the gap can be ignored as the volume of the air passing throuph it is very small.

The design of a suitable fan is of primary importance for effective heat dissipation. The fan is considered to generate a certain water guage head and various head losses in the interval circuit in the fan are then deducted from the generated head to obtain the avaiblable head. The various special constructional features of the fan are considered as a means of reducing the internal drops. The centrifugal type is more often used as propeller type do not generate sufficient pressure at the usual speed of motors. Formula are available for generated head and losses of different types of fans. (J.11)

Causes of air pressure drop. (J.7).

Cause	Formula for Pressure drop in inches of water guage	Example
Sudden contraction	$0.031\ V^2$; V = Velocity after contraction.	Inlet to ducts
Sudden Expansion	$0.062\ (V_1 - V_2)^2$; V_1 = velocity before expansion; V = velocity after expansion	Outlet from ducts
Sudden bend	$0.062\ V^2$; V = velocity at bend	Passage from axial to radial duct.

The outlet loss, inlet loss and friction loss are the head losses in a fan. The outlet loss relates to the stored energy in the moving air discharged into the atmosphere. A diffuser is used to reduce the loss in which air from the fan is gradually slowed down by providing an increasing cross section of path. This is a diffuser. A volute is sometimes used to collect air direct from the blades but is not very effective in conjunction with diffuser. In electrical apparatus the fan is embodied in the machine and the scope for employing diffuser is limited. Inlet loss is due to the turbulence at impact of the air with the blades, just as the particles of air become caught up by the blading. The method to reduce this loss is to curve the blade at entry to be parallel with the normal direction of air flow and then giving a gradual change of direction. In take loss is high in high speed. Small diameter fans. Even with very good condition intake cannot go below 20% of velocity head at intake. If h is the pressure drop in inches of water guage across the machine excluding the fan the air speed will be about $25\sqrt{2240\ h}$.

There are certain limitations on fans for electrical machines. In low speed machines upto 2' diameter it is not practicable, to have a fan larger than the yoke diameter though the mechanical stress on blades would permit larger fans. Also the inside diameter of the fan is fixed by the back plate and shroud diameter. Thirdly curved blade fans cannot be used in reversing machines and only radial blade type can be adopted.

Ralph Poore has discussed (J.8) how two centrifugal fans in series permits greater efficiencies on variable load than in parallel. In this system each fan feeds one half of the machine at full out put. In case of breakdown of one fan the other can still be used. There is also argument in favour of the series system by rotating the fans in opposite direction.

9.6 An equivalent thermal circuit for non-ventilated Induction Motors. (J.5)

In the past heating problems were usually solved by adding more active material to the machine, by increasing the dissipating area by means of fan, however, in recent years the demand has been for more HorsePower per pound of material. In addition, industry in general is beginning to look with disfavour upon the high Noise level inherent with bigger fans. Under these circumstances, it is imperative that our techniques for predicting motor heating be greatly improved.

The method is an equivalent thermal circuit based on the similarity of ohms law of electrical conduction and Fourier's law of heat conduction. To be simple, this deals with the temperature rise of totally enclosed non-ventilated squirrel cage Induction Motors by means of an equivalent thermal circuit. This covers normal slip machines having no effective rotor or stator ventilating ducts. This is for steady state heating condition.

This circuit represents the thermalcircuit of the

entire machine rather than certain positions of the motor. Each thermal quantity will represent a source of watts loss in the motor, and the generated potential across each generator will be a function of the generated losses.

Equivalent Thermal Circuit:- (J.5)

The basic equation for the circuit is $T = WR_t$.

When R_t - the thermal resistance of the element.

W = the rate of flow of heat energy through the element in Watts.

T = the thermal potential causing the flow of heat energy when the flow of heat is Pure.

Conductor phenomena

$$R_t = \frac{1}{K_t} \quad \frac{L}{A}$$

Key to symbols in the figure.

——○—— generator representing motor heat losses

——wwww—— Thermal resistance.

Complete thermal equivalent circuit of the totally enclosed non-ventilated induction motor is shown in figure. The thermal generators in the figure represent the following:

1. Core Iron
2. Tooth Iron
3. Coil head stator copper
4. Slot stator copper
5. Windage
6. Rotor end ring.
7. Rotor Bar
8. Bearing fricting.

BND Res = Boundary resistance where heat must be transferred from a surface to air or vice versa.

The simplified thermal circuit is shown in the next figure. This circuit containing 8 thermal generators is completely explained in the figure. Although the circuit is somewhat involved, the solution of the circuit itself is not a real problem; rather it is the determination of the constant which present the real problem. As a whole, those resistances, representing pure conduction can be determined relatively simple. Even if the constants are known with fair degree of accuracy it is improbable that the designer would make frequent use of the circuit unless a d.c net work analyser or a degetar computar was available.

CHAPTER X.

Rating of Induction Motors.

10.1 Introduction. The Importance of an adequate system of rating for industrial motors can hardly be overemphasized. The rated horse power of a motor is a measure of its working ability, for whose integrity the entire electrical industry is responsible. The name plate rating implies a host of different qualities built into the motor, including overload and starting ability, temperature endurance, high potential strength and other matters covered by national standards. For the economic use of motors, the fair comparison of competitive designs, the maintenance of a proper and not excessive variety of types, the intelligent handling of power supply and control problems, and for many other reasons, it is essential that American Standards of rating convey a definite guarantee of balanced characteristics and quality in motor design.

The essence of the rating problem is to find a simple test procedure that will uniquely define the output limitations of the apparatus in question. The outputs of gas engines, steam locomotives pumps, turbines and other mechancal apparatus are limited by mechanical considerations. Their continuous output ratings are, therefore, very little below their maximum momentary capacities, and users do not expect to load them appreciably beyond their ratings, even momentarily. On the other hand, the output of a transformer

is limited almost entirely by thermal considerations the theoretical point of maximum output with a constant voltage supply being far beyond the safe thermal limit. Hence, transformer users may permit high short time overloads, so long as prescribed temperature limits are not exceeded.

Electric motors are subjected to mechanical as well as thermal output limitations, both of which must be recognised in a practical rating system. The thermal limits are controlling in continuous operation, with present insulating materials, so that the close similarity between motor and transformer methods of rating that has always existed is entirely logical. In many cases, however, such as hermetic refrigerator motors, responsibility for cooling is entirely in the user's hands, so that usual temperature rise guaranteed will not be made by the motor manufacturer. Future trends will, therefore, assuredly require a rating system based on torque ability alone.

10.2 History.

A thorough discussion of the motor rating question before the Institue and by the entire industry by about 1925 resulted in the adoption of the present American System of a single continuous rating with 40 degrees centigrade rise for general purpose motors. The conclusions were (J.1).

1. Any basis of rating is at best merely an arbitrary designation of size. It is merely one of many that might be chosen.
2. Rating alone is insufficient and must always be supplemented by a clear definition of the service conditions

for which the rating is chosen. In fact, determination of the usual service conditions must necessarily precede the determination of a suitable basis of a rating.

3. Rating alone is an insufficient indication of the inherent ability of the motor to perform satisfactorily under service conditions and duty cycles differing from the usual. It is merely one indication of size and must be supplemented by other service information to permit the intelligent selection and economic application.

The following were the recommendations made (J.1)

1. The division of industrial power motors into 2 classes with the dividing line at 200 horse power and in each class:

2. A normal continuous-duty single rating for the open-type motor as the standard designation of size.

3. Well defined usual service conditions for the normal rating.

4. Service information showing permissible loadings under other duty cycles or other service conditions different from the usual service conditions.

5. Specialized motors with special ratings only where the performance characteristics required or the nature of the duty cycle do not permit of applying a service factor to the normal rating of the standard motor.

Four of these five recommendations were carried into effect in the AIEE and National Electrical Manufacturers Association standards and experience since has well justified this action. Mr. Collens' fourth suggestion, however, that

information should be prepared, showing permissible loading of standard motors under other duty cycles or unusual service conditions, has never been adequately carried out nor incorporated in the industry standards.

The progress of the art between 1929-39, inclding the development of the modern automatically controlled cyclic loads of air conditioning and refrigeration, has brought a tremendous increase in number and variety of motor application. The recent trend has been to develop many spedial motors, each adapted to drive a particular piece of mechanical equipment, often with overload and starting abilities much in excess of those normally associated with therі name-plate continuous ratings.

The objectives of revised standards should be to specify a standard type of motor adapted for the greatest variety of applications and to facilitate the economic use of the full capacity of the motor under all service conditions.

10.3 <u>The Present Rating System.</u>

Present American Standards provide for two braad classed of continuous-rated motors (J.2)

General-Purpose motors (200 horse power or less and 450 r.p.m. or more) have a single continuous rating but must be suitable for carrying 115 per cent of rated overload continuously under usual service conditions, with the ambient temperature 40 degrees centigrade or lower. These motors are offered in standard ratings for use without restriction to a particular application. They are required to meet the low limiting temperature rise of 40 degrees centigrade by

thermometer at rated load, to allow a greater factor of safety where the service conditions are unknown.

Special purpose motors specially designed for a Particular Power application where the load requirements and duty cycles are definitely known, have a single continuous rating of 50 degrees centigrade rise by thermometer, without any continuous overload requirements.

The standards also specify minimum values of starting, pullup, and breakdown torques for each type and class of motor.

These provisions should undoubtedly be retained, but it appears desirable to add to or modify them in four respects;

First, the standards should include operating recommendations for general purpose motors in intermittent or varying load service and in different ambient temperatures, so that the inherent overload capacity of the motor can be safely utilized. This is in accordance with the American Standards for transformers.

Second the general use of small motors in intermittent rather than continuous service should be recognised in the standards by requiring relatively greater starting and breakdown torques and greater temporary overload capacities than for large motors. Unless such provisions are made, off standard motors will be used to an increasing extent and control problems will be complicated, to the detriment of the public as a whole.

Third, the more general use of various protected motor designs suggests that their allowed temperature rises be reviewed and their ratings be made more nearly comparable with general purpose motors.

Fourth, the more complete utilization of motor overload capacities, implied by this program, should be accompanied by more exact determination of insulation temperature, In many modern designs, especially of protected motors, the windings are quite inaccessible, and thermometer readings on exposed parts do not accurately measure hot spot temperatures. It appears desirable, therefore, for the standards to require that stator winding temperatures be measured by resistance. (J. 3)-

10.4 Overload capacity of Standard General Purpose Motors.

Assuming adequate mechanical strength, the measure of a motor's momentary overload capacity is the adequacy of its torques, giving assurance that the motor can bring the load to speed and carry it under low voltage high friction, or other unforeseen temporary conditions. American standards now require that general purpose Polyphase Induction Motors shall have a breakdown torque of not less than 200 percent. Allowing for ten percent reduction in voltage and 20 percent margin for variations in individual conditions loading , this 200 percent breakdown torque will enable loads not over 135 per cent of the rating to be carried sucessfully, subject to heating limitations.

Therefore, under the present standards, motors cannot

be relied upon to carry momentaryoverloads of more than 35 per cent in excess of the rating, under a reasonable variety of service conditions. In practice, designers normally provide more breakdown torque than required by the standards especially for the smaller and higher speed motors, so that many designs can carry considerably greater short time overloads.

The starting current of a large Poly Phase Induction Motor is almost directly proportional to the maximum or breakdown torque. The starting current is therefore, an excellent measure of the short time overload ability.

With the adequate torque margins the remaining important factor in overload capacity is the temperature rise. This must be low engugh to ensure adequate life under the expected overloads. While there may be other objections to high temperatures in special cases, the chief purpose of limiting the standard temperature rise to protect the public from the inconveninence and loss that would be occasional by motors with a short insulation life. As the actual life that a motor will have under a given temperature cannot be determined by acceptance tests alone, it is peculiarly important that the standards provide for safety in this respect.

When and if ~~a~~ insulating materials of greater temperature endurance come into use, they may be utilized to permit reduction ~~a~~ in motor size, with a higher continuous rise. It seems desirable, however to use such higher temperature matierals and limits first on totally enclosed machines, permitting interchangeable dimensions with open type motors of present temperature limits. For open type motors with liberal overload torques, and reasonable efficiencies, it costs

thermometer will normally have an actual hot spot temperature of not over 102 degrees centigrade, when operated continuously at 115 per cent of the rating, in accordance with the standard service factor, indicating about ten years' useful life. Since, in practice, the average ambient temperature in the United States is gnerally below 30 degrees centigrade, the typical motor operating continuously at 115 per cent of its rating will have an average hot spot temperature of about 90 degrees centigrade, giving an indicated useful life of roughly 25 years. When motors are operated below their ratings considerably larger insulation life may be expected.

If, however, a motor is employed an intermittent service, with short time overload periods repeated at int rvals of hours, days or longer, and periods of complete idleness between, the actual life of the motor at the same loading may be considerably longer, temperature alone considered. Therefore, reasonably higher temperatures may be permitted on intermittent service. (J.5)

We shall assume that whether a motor is operated at a given high temperature one month in every ten, or an any other cycle with the motor idle nine-tenths of the time, the insulation will deteriorate at the same average rate, giving the same total years of life. We shall assume also that the temperature life curve is a constant exponential curve and the temperature variation are not accompanied by other deteriorating conditions, such as variable dirt, or

for the desired intermittency always occur at the 115 per cent load point on the standard motor characteristic curves, giving the same torque chracteristics as in continuous operation at the original 115 per cent service factor rating.

If the equivalent circuit constants of the motor do not change from increased magnetic saturation or other cause, and all fixed losses increase as the square of the volts per turn, all the chracteristic curves of the motor will retain the same shapes.

The per unit output of the redesigned motor is obtained by multiplying the output scale for the original motor by $1/a^2$, where a is the ratio of the new to the old number of winding turns, the new full load characteristics being the same as at a load a^2 on the original motor. The rated load in horsepower is assumed to remain the same in all cases.

By this process, the efficiency and, therefore, the total losses and temperature rise at rated load will be slightly changed from their original values. A limit is set on the stepping up of the torque with reduced winding turns by the excessive increase of the no load current when the magnetic flux density is increased byond the saturation point. For low speed motors particularly, there is a definite value of volts per turn of winding beyond which a further increase will reduce instead of increasing the breakdown torque, for fixed magnetic dimensions. A further limitation is set by the rapid increase in full load current and temperature as the maximum

EFFICIENCY point is brought beyond the point of rated load.

10.7. Rating of Protected Motors.

The general use of protedted motors with different degrees of enclosure suggests that their permissible overloads should also be determined. Present standards allow 50 degrees centigrade rise by thermometer, for splash proof motors, and 55 degrees centigrade for totally enclosed and fan cooled designs with a service factor of 1 instead of 1.15. The extra 5 degrees centigrade for enclosed motors is generally understood to be allowed because of the smaller difference between the hot spot and the measured temperature than in open motors. The standards imply, therefore that all fully protected motors will have a hot spot temperature rise at rated load of 65 degrees centigrade, or practically the same as that of the standard general purpose motors of the open type at 115 per cent load.

Hence the curves for the general purpose motors apply equally well to enclosed motors if the actual loads are divided by 1.15 or if the rating of the enclosed motor is taken as 87 per cent of the name plate value.

It is very desirable to build protected and open type motors in the same frame size and with interchangeable characteristics, and it may be urged that the exclusion of dirt, excessive moisture, and other protection to insulation in enclosed motors justifies a higher temperature for the same service. It appeears probable that new insulating matesials may permit this in future, but further operating ex-

experience records should be obtained before the standards are changed in this respect.

In usual enclosed motor designs, fewer winding turns and larger magnetic dimensions are employed than in open motors, to reduce the copper losses and temperatures. Hence, such motors normally have a little higher breakdown torque and starting current than open motors, and they are even better adapted to carry short time overloads. For this reason, and in view of the presumably longer insulation life at a given temperature because of moisture and dust exclusion, it is suggested that service factors be applied to enclosed as well as open motors. While no specific recommendations are now offered, it is clear that the inherent ability of an enclosed motor to deliver short time overloads will be utilized in the long run, and standards on operating recommendations should be prepared to facilitate this.

The cooling motors for hermetically sealed refrigerators, enclosed gas pumps and other built in applications, is entirely in the control of the user. Temperature rating standards do not apply to such motors, therefore, and the operating temperature may be anything that the user's experience justifies. By adhering to the proposed breakdown torque and starting current rules, and normal efficiency values, however, assurance is given that the motor will have the torque ability represented by the name plate horsepower. With improvements in insulating materials such torque and starting current rating methods may be expected to supersede the temperature system to an increasing extent.

10.8 Classification of Insulating Materials.

Note: The following classification is made on that adopted by the International Electrotechnical Commission in 1935, but is amplified to take account of recent developments in insulating and impregnating materials.

Classification of Materials: For the purpose of this standard insulating materials are classified in the following ways:

Class O. Not applicable to this standard.

Class A. Cotton, Silk, Paper, and similar organic materials when impregnated or immersed in oil, also substances known as enamel, oboresin and synthetic resin coatings applied to conductors.

Class B. Mica, asbestos, glass fibre and similar inorganic materials in built up form with organic binding substances. A small portion of Class A materials may be used for the structural purposes only. Class H, Mica, abbestos, glass fibre and similar inorganic materials in built up form with binding substances composed of silicone compounds in rubbery or resinous forms or materials with equivalent properties. Such matérials will permit safe working temperatures higher than are permissible for class B insulation. For the time being they are broadly covered by the designation "Class H" for which working temperature limits have not yet been fixed. When they became generally available and their limitations are established the standard will be approrpriately extended.

A minute proportion of Class A materials may be

used only where essential for structural purposes during manufacture.

Class C. Mica, without binding materials, porcelain, glass, quartz, and other similar materials.

10.9 Temperature Measurements.

Recongised method of measuring temperature.

Three methods of measuring temperature are recognised by this standard:-

Thermometer Method

Resistance Method

Embedded temperature detector method.

Thermometer method.

When using method, the temperature shall be measured by thermometer applied to the hottest accessible surfaces of stationary parts during the test period and by other thermometers applied to the accessible surfaces of rotating parts as soon as the machine is stopped after the test.

The term ' Thermometer ' means mercury or alcohol bulb thermometers but at the option of the manufacturer may also include thermocouple and resistance thermometers when applied to the surfaces accessible to bulb thermometers.

In all cases the bulb of the thermometer, except at the point of contact shall be covered with a pad of felt, cotton wool or other non-conducting material 1/8in. thick extending at least 3/4in. every direction from the bulb and pressed into contact with the surfaces to which this

is applied, to prevent loss of heat by radiation and convection from the bulb. (J.7).

Resistance Method.

In this method the temprature rise of the winding shall be determined by the increase in the resistance of the windings. However, if desired a check may be made by thermometers applied to the accessible surfaces of the windings and the temperature rise thus determined shall not exceed the permissible limit specified for the resistance method.

The temperature of the windings as measured by the thermometer before commencing the test should not, differ from that of the cooling medium. The initial resistance and initial temperature of the windings shall be measured at the same time.

Since the resistance of copper over the range of temperature referred to in this standard varies in direct proportion to the temperature above minus 234.5°C, the hot temperature is obtained from the following formula:

$$t_2 = \frac{R_2}{R_1} (t_1 + 234.5) - 234.5$$

R_2 = Resistance of winding hot

R_1 = Resistance of winding cold.

t_2 = Temperature of windings hot, o.c

t_1 = Temperature of windings cold oc.

Precautions to be observed in making resistance measurements:

Since the increase of resistance of copper for 1 centigrade degree change in temperature is only about 0.4

percent special care must be taken when measuring the temperature of windings by this method to ensure that apparatus and methods of adequate accuracy are employed. In machines whose temperatures may change very rapidly after shutting down (e.g Water cooled machines) adequate rapidity of measurement is also necessary.

To determine the temperatures of the windings when hot, an accurate measurement of resistance and associated temperature must be taken when the windings are cold. Special precautions should be taken in the case of large mechines in view of the inaccessibility of the windings and the possiblity of unequal temperatures in different parts of the machines. These cold readings should, therefore, be taken after the machine has been standing for sometime, so that it may have assumed the temperature of the surrounding atmosphere. In the case of larger machines this length of time may quite well be 24 hours or larger, and in engine rooms which are subject to fluctuations of temperature, further precautions will be necessary.

In the measurement of rotor winding resistance the brush contact resistance is appreciable and should be eliminated. In order to do this a speical brush may be used which is s insulated from the rest of the brush gear and serves only to carry the small voltmeter current from the surface of the slip ring. (J.8)

<u>Embedded Temperature detector method.</u>

Embedded temperature detectors are thermocouples

built in the machine during construction at points which are inaccessible when the machine is completed. This terms does not include the necessary measuring instruments.

When the internal temperature of a machine is to be measured by embedded temperature detectors, at least six detectors, suitably distributed around the stator, should be built into the machine. All reasonable efforts consistent with safety shall be made to place the embedded temperature detectors at the various points at which the highest temperatures are likely to occur. At least three detectors shall be placed between the upper and lower coil sides within the slots and midway between radial ventilating ducts, if any.

Time at which temperatures are to be taken.

The temperatures of a machine shall, whenever possible, be taken during working as well as after stopping the machine, the highest temperature thus obtained shall be adopted. When successive measurements show increasing temperatures after shut down, the highest value shall be taken.

If the initial interval between the instant of cutting off the Power and the machine coming to rest is considerable, corrections, by means of extrapolation of a cooling curve, shall be adapted so as to obtain as nearly as practicable the temperature at the instant of shut down (J.9).

10.10. European Standards of Motor Rating.

Comparison of French, British, and German with

American standards reveals considerable differences in both torque and temperature limits for general purpose. a.c motors. The quantity of work performing ability measured by one continuous horsepower of motor rating cannot be accepted as an international standard, therefore but must be evaluated by reference to the customs in the country of origin. (I.6).

All four countires give general purpose industrial motors a single continuous rating, limited by temperature rise. The first three countries allow a maximum ambient temperature of 40^{o} centigrade, but the German allow 35 degrees centigrade ambient. American and British rules for open type motors specify 40 degrees centigrade rise by thermometer; while French rules specify 55 degrees centigrade by resistance, or 50 degrees centigrade by thermometer if resistance measurements are not practicable, and german rules specify 60 degrees centigrade by resistance or thermometer, whichever gives the higher reading. In all cases temperatures are measured before and after shut down, the highest reading being taken and no tolerances from the guaranteed values being allowed.

For these open type low voltage motors, it is fair to say that the hot spot temperature rise is not more then 25 per cent above the thermometer reading 12 per cent above resistance measurements. On this basis, the hot spot temperature rises of the four motors at rated load will be 50 degrees, 50 degrees, 62 degrees, and 67 degrees centigrade respectively. In continuous operation at full load in a

40 degree centigrade ambient, therefore, the American and British motors will have a life expectancy of 25 years, the French motor 10 years, and the German motor 6 years, assuming the same design margins over guarantees. American special purpose motors, or general purpose motors operated at their 115 per cent service-factor rating, however, have the same temperature rise and the ten-year life expectancy, of the French motor. In a 35 degree centigrade ambient, these periods would all be increased about 50 per cent.

For totally enclosed motors, the American rules allow 55 degrees centigrade rise by thermometer without any service factor, and the British allow 50 degrees centigrade while the French and Germans keep the same 55 and 60 degrees rises by resistance allowed for open motors. In a 40 degree centigrade ambient with continuous operation at rated load, these correspond to life expectancies of five, ten, ten and six years respectively, the American having the shortest life in this case.

The five degrees spread between open and closed motors under the American rules may be justified technically on the grounds of a lower hot-spot differential, slower deterioration of insulation from other causes than temperature in enclosed motors, and the frequent use of these motors in outdoor installations at lower ambients. It is economically sound, because it is relatively a great deal more expensive to lower the temperature of a fully enclosed motor than of an open motor, and because it facilitates interchangeable

mounting dimensions for open and closed motors.

The inherent, or short term capacity of a motor is determined by its maximum torque, which is quite independent of the temperature rise. For a true comparison of an induction motor's ability to handle all sorts of loads therefore, it is necessary to know its breakdown torque.

The American and British rules both specify a minimum of 200 percent breakdown torque, or 100 percent overload torque, for standard General-Purpose Industrial Motors. The French rules require only 150 percent, and the German rules 160 per cent breakdown torque, on the basis of continuous ratings; The french rules recognise seven, the British two, and the German four kinds of intermittent or short time ratings, for which 200 percent breakdown torque is required by all except the French rules. None of the rules prescribe any starting current limits for industrial motors.

The breakdown torque values, which are the best measure of magnetic dimensions and mechanical ability, indicate that the standard American, British, French, and German continuous-rated motors have relative sizes of 100, 100, 75 and 80 respectively.

The American, British, and French all have special rules for fractional horse Power motors, which cover motors smaller than one horse Power at 1500 r.p.m. one horse power at 1000 r.p.m. and 600 watts, respectively. American rules

prescribe the same breakdown torque and temperature limits as for larger motors, except that 175 per cent breakdown torque is required for single phase motors. British rules require only 125 percent breakdown torque for Poly Phase add 100 percent for single phase motors. They also extend the 50 degree temperature rise limit to include drip-proof as well as totally enclosed motors, keeping 40 degrees centigrade for open motors. French rules provide a special "domestic" service rating for fractional motors, whichis defined as equivalent to continuous service at two-thirds of the name plate horse power, implying a breakdown torque . 225 percent of the continuous capacity. The French also provide a high temperature rise for all fractional horse power motors, 65 degrees centigrade for continuous rated motors, and 85 per degrees centigrades for domestic motors. for temperate climates, the latter on the basis of a maximum ambient, temperature of only 20 degrees centigrade.

It is thus evident tht continental standards call for materially smaller motors for a given continuous rating than American and British standards.

R. Langlois-Berthelot has written a comprehensive article on the temperature life of electrical machines, including an historical summary of the subject. His views on the normal life of the insulation are summarized in the following statements:

We shall take ..... the (hot spot) temperature) Θ (fr rating purposes) as that permissible in a machine to

assume in practice a normal life of 15 to 20 years and we shall assume as a fact of experience that this normal life is equivalent to continuous operation for 2 years, or 17000 hours, at the maximum temperature; so that .... θ will correspond on the insulation life curve to a life of two years. These are the average normal conditions which take account of the usual variation of ambient temperature and of load. This interpretation of the temperature θ conformx to experisnce with machines and to the opinion of known experts who have been willing to express their views.

This point of view that the standards ought to set temperature limits to give long life under average conditions of reduced load and reduced ambient appears fundamental in continental standards, in strong contrast to the American view point that the rated temperature should be low enough to assure a long life in rated load operation at the maximum ambient temperature.

CHAPTER XI.

D E S I G N.

INTRODUCTION:

The Motor with a high capital cost may have a low running cost due to the increased efficiency resulting from the better qualities of the materials used. A machine may have a low capital cost and a high running cost. Theoretically any increase in the efficiency (and perhaps also the Power factor) is of financial advantage to the user of a motor, and the capitalized value of the annual saving on the cost of the Power is available for the additional cost of a motor designed more liberally to secure the increase. So the designer must adopt a via media so that the user gets the maximum economy both considering the interest on the capital cost and the running Power cost. Then only the design can be a prospective competitive one. So the process of design is to obtain the dimensions and electrical particulars of a machine to satisfy a given specification covering horse Power, speed, temperature rise and condition of service. For standard Motors the designer's task is considerably lightened by routine methodsof using tabulated data giving the approximate values of quantities that would otherwise have to be calculated. The trend of a design should be towards greater and greater output from a given mass of materials, and this can only be affected by scientific design and proper proportioning of the machine.

The formula for effective value of induced Voltage is

$$E = 4\ Kf\ Kp\ Kd\ c\ f\ \phi \times 10^{-8} \quad \text{Volts} \qquad (1)$$

If the hypothetical total flux $\phi_t = \frac{\phi p}{fd}$

$$\phi = \frac{E \times 10^8}{4\ Kf\ Kp\ Kd\ cf}\ 10^8$$

$$\text{and } \phi_t = \frac{E\ p\ 10^8}{4\ f\ Kf\ Kp\ Kd\ c\ \ fd.} \qquad (2)$$

If N is the no. of conductors No. of Coils $C = \frac{N}{2}$

$$\text{and} \quad f = \frac{n\ p}{120}$$

$$\phi t = \frac{E \times 60 \times 10^8}{n\ N\ fd\ \ f\ 4\ Kp\ Kd\ kf}$$

The form factor Kf, the distribution factor Kd flux distribution factor fd depend only upon the number of slots and shpae of air gap flux distributed curve. Those factors can therefore be combined into one factor called winding constant.

$$Cwm = Kf\ Kd\ fd.$$

$$\phi t = \frac{E \times 60 \times 10^8}{n\ N\ \ kp\ \ Cwm.} \quad \text{lines} \qquad (3)$$

Output horse Power

$$= \frac{VI \times eff \times \text{Power Factor}}{746} \qquad (4)$$

Total flux ϕ_t = II Dl Bg.

IN fc = II D g.

Where Bg = Specific magnetic loading

g = Specific electric loading

E = 0.95 applied Voltage V

$$h.p = \frac{II\ D\ lg\ Bg\ II\ D\ gn\ \ Cwm\ \ eff.\ \ pf.}{44.75 \times 10^{11}}$$

$$\frac{D^2\ ln}{h.p.} = \frac{4.54 \times 10^{11}}{Bg.\ g.\ cw.\ eff \times pf.} \qquad - (5)$$

$$\frac{D^2\ \ ln}{Watt\ Output} = \frac{4.54 \quad \times 10^{11}}{7.46 \qquad Bg.\ g.\ cw\ eff.p.f.}$$

Equation (6) can be taken as:

$$\frac{D^2\ ln}{Watts\ Output} = Co$$

Where Co = Out Put co-efficient.

11.2. SPECIFIC LOADINGS:

In an Induction Motor Bg. directly influence the core loss and magnetising current as this has an important effect on the Power factor.g determines to a large extent I^2R loss. The flux density in the air gap generally lie between limits 25000 and 45000 lines/ Sq.in.

Proper winding distribution will reduce harmonics of low order so that their effect will be small. The number of

turns per pole may be distributed according to Sine Law.

The winding distribution factor for concentric type is weighted mean chord factor and is calculated by multiplying the chord factor of each coil per pole group by turns in the coil and dividing the sum of these products by the total number of turns.

The Capacitor start Motors requires an auxiliary winding in the Stator or addition to main winding. This winding is provided at 90^{o} with the Main winding and has a smaller conductor. The auxiliary winding can be arranged and distributed as explained for the main winding.

11.3. ACTUAL DESIGN.

Design calculations of ½ h.p. Capacitor Start Single Phase Induction Motor.

11.4. SPECIFICATIONS.

Full load output = ½ h.p. Supply Voltage 230 Volts at 50 cycles. The full load efficiency and Power factor should not be less than 65% and 60%. The Motor must have a locked Motor torque at rated, value not less than 200% of full load torque, with a locked Motor current not over 35 amperes. The temperature rise is not to be above 40^{o}c for continuous duty and full load Speed not

less than 1425 rpm.

The machine is to be designed for the given Standard Stampings.

11.5. Main Dimensions:

Watts Out Put = 371 Watts.

The number of Poles for 1500 Synchronous r pm = 4.

$$\text{Watts / r p m} = \frac{371}{1500} = 0.248$$

The output co-efficient Co fromfigure is 200

$$D^2 L = \frac{200 \times 371}{1500}$$

Bore diameter = 3.5"

$$\text{So } L = \frac{200 \times 371}{1500 \times 3.5^2} = 4.0"$$

11.4. Main Winding:

Teeth Width = 9/64"

Assuming Stacking factor of 0.95

$$\text{Total tooth Section} = \frac{9}{64} \times 4.25 \times 28 \times 0.95$$

$$= 16 \text{ sq. inches.}$$

For a flux density of 105,000 lines 10" in the tooth.

$$\text{Total flux } \phi t = 105,000 \times 16 = 1.68 \times 10^6 \text{ lines.}$$

Assuming a value of 0.795 for cwm turn equation (2)

total number of conductors in series for main winding.

$$Nm = \frac{p\ E\ 10^8}{2\ f\ cwm\ \ fd.}$$

Where E can be assumed to be 0.95 applied Voltage.

$$Nm = \frac{4 \times 230 \times 0.95 \times 45 \times 10^8}{50 \times 1.68 \times 10^6 \times 0.795 \times 0.637}$$

$$= 920 \text{ turns.}$$

$$\text{The turns in series per pole} = \frac{920}{2 \times 4}$$

$$= 119 \text{ or } 120.$$

Total number of Conductors $= 120 \times 8 = 960.$

The readjusted Value of Total flux for changes in winding constant and conductor is

$$= 1.68 \times 10^6 \times \frac{920}{960} \times \frac{0.795}{0.8} = 1.6 \times 10^6$$

flux density in the tooth = 10000 lines/□″

$$\text{Line Current} = \frac{1}{2} \times \frac{746}{0.65 \times 0.6 \times 230} = 3.9\ A$$

Assuming a current density of 3000 amp / sq. in area of

$$\text{Stator conductor is} = \frac{3.9}{3000} = 0.0013.$$

From wire tables nearest gauge is 19 S.W.G. and its area of cross Section = 0.0012569.

To obtain a high value of space factor enamelled wires were used.

The Slot area is 0.166

Space factor in the Slot in which no. of conductor is maximum is 53 Conductors.

$$= \frac{53 \times 0.0012569}{0.166} = 0.4$$

Stator Yoke.

Stator Stamping has 28 Slots and yoke depth is 3/8"

$$\text{Flux/ Pole} = \frac{1.6 \times 10^6 \times 0.637}{4} = 2,56,000 \text{ lines}$$

Yoke Depth = 3/8"

Active iron area of cross section of Stator yoke

$$= \frac{3}{8} \times 425 \times 0.95 = 1.5 \text{ sq. in.}$$

$$\therefore \text{Yoke Density} = \frac{256000}{2 \times 1.5} = 85,000 \text{ lines /sq.inch.}$$

11.7. ROTOR.

Shaft Diameter 3/4". Core depth 5/8"

20 rectangular Slots.

$$\text{gap length lg.} = .005 + .00035\,D + .001L + \frac{.003V}{1000}$$

Where V is the perepheral speed in feet/ See

$$= .005 + .00035 \times 3.5 + .001 \times 4.25 + .003 \times 1.37$$

$$= 0.01457 \text{ inches.}$$

Taking gap length is 0.0145 "

Rotor diameter = 3.5 - 0.029 = 3.471 inches.

Rotor bar is rectangular Section 5/12" x 5/16"

If ab is cross sectional area of bar, a_r is cross sedtional area of ring and I_b is the r.m.s. current in the bar N2 No. of bars R.M.S. current in the Ring.

$$Ir = \frac{N2}{2\ p} \quad x \quad \pi \quad x \quad \frac{\sqrt{2}}{\sqrt{2}} \quad x\,2\,I_r \quad = \frac{N2 \quad Ib}{\pi\ p}$$

For same current density:

$$\frac{I2 \quad Ib}{p\pi\, ar} = \frac{Ib}{ab}$$

$\therefore$ area of c.s. of ring $= \frac{N2\ a}{\pi\ p} = \frac{20\ x\ .049}{\pi\ x\ 4}$

$= 0.078$ sq. in.

So copper ring section can be = 5/10 x ¼

Outer diameter of ring = 3.47 - 2 x 5/16 = 2.846 "

Inner diameter = 2.846 - ¼ x 2 = 2.346"

Width of rotor teeth at 1/3 ht. fromroot of teeth

$$= \frac{\pi\ x\ 2.846}{20} - \frac{5}{32} = 0.29$$

Rotor length = 4.5 "

Stator leakage factor = 0.95

Sectional area of rotor tooth = 20 x .29 x 4.5 x .95

flux in the tooth = 1.6 x 10^6 x .95

Rotor tooth density $= \frac{1.6\ x\ 10^6\ x\ .95}{.29\ x\ 4.5\ x\ .93\ x\ 20}$

= 63,000 cms / sq. inch.

Air gap density $= \dfrac{1.6 \times 10^6 \times 0.95}{\pi \times 3.471 \times 4.25}$

$= 38,600$ lines /sq.in. (38,600)

Rotor core radial depth = 5/8 inches.

Rotor Core density $= \dfrac{256000 \times .95 \times 9}{2 \times 5/8 \times 4.5 \times 93}$

$= 42,000$

Ratio of Stator Slot opening to air Gap length

$= \dfrac{1}{16 \times .0145} = 4.3$

Carter's co-efficient $\delta = 2.15$

$\therefore$ apparent co-efficient to take into account the

effect of teeth is $Ks = \dfrac{Y\ is}{wt + d.lg.}$

$= \dfrac{3.94}{.0625 + .0312} = 1.14$

Total gap co-efficient $= 1.14$

So virtual airgap length $= Ks \times lg$

$= 1.14 \times 0.145$

$= .0165$

11.8 Motor Constants:

Stator winding resistances:

The length of half mean turn for centimetr of the

coil per pole of a concentric type winding:

$= \dfrac{4.2\ (D + ds)}{Ss}$ Slot spanned + 1

Where ds = depth of slot.

For the winding distribution shown length of 2 half mean turn is

$$\left(\frac{4.2.\ (3.5 + .53)}{28} \times 2 + 4.25 \right) 24 = 131$$

$$(.605 \times 4 + 4.25)\ 43 = 287$$

$$(.605 \times 6 + 4.25)\ 53 = 417$$

$$835$$

Length of mean turn $Lsm = \frac{835}{120} = 7$ Resistance of 19 gauge wire from 1000 yards = 19.439 ohms.

Resistance of main winding at 75° C

$$R1M = \frac{7 \times 960}{36} \times \frac{19.439}{1000} \times 1.2 = 4.35 \text{ ohms.}$$

Total length: = 186 yards.

Rotor Resistance:

Resistance of Sq. cage Rotor is calculated as follows:

$$lb = 4.5 + .625 + .25 = 5.5''$$

Total Resistance of Sq. cage bars $= \frac{16\ N2\ r}{10^6\ ab.}$

When r is specific resistance

wt of two end rings $= \frac{2 \pi\ Der}{10^6\ as.}\ r$

Total resistance of squirrel cage winding is equal to total copper loss by current squirred and is

$$= \frac{6\ b\ N2\ R}{10^6\ ab} + \frac{N2^2}{\pi^2 p^2}\ \frac{2\ \pi\ Der}{10^6\ ar}$$

$$= N_2^2 \left(\frac{lbr}{10^6 ab\ N2} + \frac{.64\ \ Der\ \ r}{10^6\ \ ar\ \ p^2} \right)$$

Squirrel Cage winding in terms of Stator winding is

$$= \frac{N1^2\ \ Cw^2n\ \ M^2\ r}{10^6} \left(\frac{lb}{ab\ N2} + \frac{.64\ \ Der}{p^2\ \ ar} \right)$$

<u>Equivalent Rotor Resistance:</u>

$$Rr = \frac{N2^2\ \ CW^2\ \ M.r}{10^6} \left(\frac{lb}{ab\ N2.} + \frac{.64\ \ Der}{p2\ \ as}\ \ K\ ring \right)$$

$$r = 0.692$$

$$m = 2$$

Ratio of Inside to outside diameter of ring

$$= \frac{2.346}{2.846} = 0.825$$

For this ratio from figure:

$$K\ ring = 0.98$$

Rotor Resistance in terms of Stator Main Winding

$$Rrm = \frac{960^2 \times .8^2 \times 2 \times 0.692}{10^6} \left(\frac{5.5.}{.049 \times 20} + \right.$$

$$\left. \frac{0.64}{4^2} \times \frac{2.596}{5} \times 64 \times .95 \right)$$

$$= .815\ (5.5 + 1.25) = 5.5 \text{ at } 25^{\circ}\ C$$

$$= 5.5 \times 1.2 = 6.6 \text{ ohms. at } 75^{\circ}\ C$$

<u>Leakage Reacåance:</u>

$$X_2 = 2\ \pi f\ N1^2\ Cwm^2 \times 10^{-8} \left(\frac{6.381}{Ss} \right.$$

$$\left. \left(Fss + \frac{Ss}{Sr.} - Fsr \right) \right) \text{ ohms.} \qquad (9)$$

Fss and Ssr are slot leakage factors are calculated from figure.

Zig Zag leakage reactance for Stator and Rotor in terms of Main winding.

$$X_z = 2\pi f \; N1^2 \; Cwm^2 \; 10^{-8} \left(\frac{2.13 \; 1}{Ss \; \; lg} \left(\frac{Wts}{4} \frac{+ wtr}{(ys + yr)} \right) \right) \text{ ohm} \qquad \text{----(10)}$$

End connecting leakage reactance:

$$Xc = 2\pi f \; NM^2 \; Cwn^2 \; 10^{-8} \left(\pi \; \phi \; \frac{D + ds}{Ss \; p} \right)$$

The magnetising reactance:

$$XM = \frac{2\pi f \; N1^2 \; Cw^2 \; 10^{-8} \; 0.625 \; 1\tau}{lg \; \; Kg \; \; p \; \; Fs} \quad .... (12)$$

Where Fs is saturation factor

Total leakage reactance of Station Main Winding plus rotor in terms of main winding of the Stator.

$$X1 + X2 = X1m = + \; Xs + Xz + Xc \qquad (13)$$

$$Xo = Xm + \frac{X1M}{2} \qquad (14)$$

$$Kp = \frac{Xo - X1M}{Xo} \quad ... \qquad (15)$$

The constant term for various reactances:

$$A = 2\pi \phi f \; N1^2 \; Cw^2 \; 10^{-8} = 1.85$$

Stator Leakage Factor: $Fss = \phi \; \frac{d1}{Ws3} + \frac{d4}{Ws\,1} + \frac{2 \; d3}{W1S + Ws2}$. as from the figure

For Stattor Slot $= \frac{WS2}{WS3} = \frac{9}{11} = 0.815$

From the Graph $\phi = 0.41$

$Fss = 0.41 \times \frac{7}{11} + \frac{1}{16} \times \frac{16}{1} \times 0$ Sime $d3 = 0$

$= 0.633 + 1 = 1.633$

For rotor slot $\frac{WS2}{WS3} = 1$

ϕ from graph $= 0.325$

$Fsr = 0.325 \times 2 = 0.65$

By equation 9

Slot leakage reactances:

$$Xs = \frac{1.85 \times 6.32 \times 4.25}{24} (1.633 + \frac{28}{20} \times 0.6)$$

$$= 4.44$$

By equation (10) Zig Zag leakage reactance.

$$Z2 = \frac{1.85 \times 2.13 \times 4.25}{28 \times 0.0145} \left(\frac{.3295 = 0.55}{4 (.392 + .55} \right)^2$$

$$= 8.5 \text{ ohms.}$$

Since station slot pitch $ys = \frac{\pi \times 3.5}{28} = 0.392$

Rotor slot pitch $= yr = \frac{\pi \times 3.5}{20} = .55$

By equation and connection leakage.

$$Xc = 1.85 \left(\frac{\pi \times 4.095 \times 4}{28 \times 4} \right) = 0.85$$

By equation (2) Pole Pitch $= \tau = \frac{\pi \times 3.5}{4}$

$$= 2.75$$

and Saturation factor of Fs = 1.14.

$$X_m = \frac{1.85 \times .645 \times 4.25 \times 2.75}{0.0145 \times 1.25 \times 4 \times 1.14} = 169 \text{ ohms}$$

Total leakage Reactance X1 + X2 = X lm

= 4.44 + 8.5 + .85 = 13.79 ohms.

$$S \quad Xo = XM + \frac{XLM}{2} = 169 + 6.9 = 175.9 \text{ ohms}$$

$$Kp = X_o - \frac{XLM}{Xo} = 0.922 = 0.96$$

<u>Core Loss:</u>

Volume of Stator Teeth = (0.394 x 1/16 x+ 9/164 + 17/132) 28 x 4.25 x .93

= 11 cubic inches.

Weight of Stator Teeth: = 11 x 0.278 = 3.06 lbs.

loss/ lb for density of 103,000 is 4 Watts. So core loss in take for fundamental flux

Wave = 4 x 3.06 = 12.24 Watts

$$\text{Volume of Stator core} \quad \frac{\pi}{4}\left(5 \quad 7/10\ )^2 - 4 - 7^2\right) \times 4.25 \times 0.93$$

Weight of Core = 5.9 x 4.25 x 0.93 x 0.278 = 6.5 lbs.

Core loss per lb. for flux density of 86800 = 2.8

Core loss in yoke = 2.8 x 6.5 = 18 Watts

Total = core loss for fundamental wave = 18+ 12.24

= 30.24 Watts.

Taking into effect harmonic fluxes.

Total care loss = 30.24 x 2 = 60.48 Watts.

Friction and windage loss will depend on this type of bearing and for all bearings it is considered to be 3.5% of output of motor.

$$\text{So friction and Windage loss} = \frac{3.5}{100}\ \frac{1}{12} \times 746$$

$$= 12.6.$$

Total no load loss: 12.6 = 60.48 = 73.08

Performance Calculation:

Operating characteristics are calculated by analytical methods prepared by Veinott.

Auxiliary Winding:

The auxiliary current is opened after rotor has attained approximately 75% of normal speed.

Starting Torque:

$$Ts = \frac{1.83\ pV^2\ a}{f}\quad RM\ \frac{(Ra\ xm - RIm\ (Xla - Xc)}{(RIM^2 + XLM^2)\ (Ra^2 + (XLa-Xc)^2} \qquad \ldots\ldots \quad (13)$$

$$Xc = Xca + \frac{Ra}{RM}\ (Zm - Xlm) \qquad (14)$$

To obtain a small capacitor and low starting current, 1.2 was selected.

No. of turns in auxiliary winding

$$N_a = \frac{K \ N1 \ cwm}{cwa}$$

Assuming winding constant of 0.85 for cwa

$$Na = \frac{1.2 \times 96 \times .8}{.85} = 1080$$

$$\text{No. of turns /pole} = \frac{1080}{4} = 270$$

For Sinusoidal distribution turns / coil are calculated.

$$\text{Winding constant:} = \frac{0.625 \times 33 + .9 \times 48 + 1 \times 54}{135}$$

$$= 0.875$$

$$a = \frac{0.875 \times 135}{0.8 \times 120} = 1.23$$

Length of half mean turn is calculated as before

$$Lat = 7.45''$$

After preliminary calculation of starting torque the 19 gauge wire is chosen.

Total length of auxiliary winding:

$$= \frac{7.45 \times 135 \times 8}{36} = 223 \text{ yards.}$$

$$Re = \text{Resistance of auxiliary winding} = \frac{53.998 \times 223 \times 1.1}{1000}$$

$$= 13.25.$$

Total Winding Resistance:

$$= 4.25 + 6.6 + 10.95$$

Rotor resistance in the auxiliary winding

$$Rra = 1.23^2 \times 6.6 = 10 \text{ ohms.}$$

Total average winding resistance:

$$= 13.25 + 10 = 23.25$$

Total leakage reactance in terms of auxiliary Winding.

$XLa = 1.23^2 \times 13.29 = 20.8$

Main Winding leakage rotor impedance:

$Zm = 13.79^2 \mp 10.92^2 = 17.6$

Rotar culvert in the main winding $= \frac{230}{17.6} = 13.08$

Copper resistance for Maximum Starting torque of eqn.(14)

$= 20.8 + \frac{23.25}{19.95} \times 3.81 = 28.9$

Capacitance in microfarod $= C = \frac{10^6}{100\pi \times 28.9} = 110$ PI

Using a Standard density of 100 p. F.

$Xc = 31.82$ ohm.

The impedence of auxiliary winding with capacitance in series.

$Za^L = Ra2 + (XLa - Xc)^2$

$= 23.25^2 + (31.82 - 20.8)^2 = 25.7$ ohms.

Locked Rotor current in auxiliary winding:

$I_{sa} = \frac{230}{25.7} = 8.95$

Current density in auxiliary winding:

$= \frac{8.95}{.00045239} = 14800$ amps / sq. in which is a reasonable value.

The locked rotor current in the line for both winding is Parallel.

$$I_L = I_{SM} \frac{(Z_M + Z_{aL})}{Z_{ac}}$$

$$= \frac{13.08\ (17.6 + 25.7)}{25.7} = 23A$$

Starting Torque:

$$T_s = \frac{0.93 \times 1.88 \times 4 \times 230^2 \times 1.23 \times 6.6}{50}$$

$$\frac{(23.25 \times 13.79 - 10.95\ \ (20.8 - 31.82)}{1095^2 + 13.79^2)\ (23.25^2 + (20.8 - 31.82)^2}$$

$$= 120$$

full load torque $= \frac{1}{2} \times \frac{33000}{2 \times \pi 1500 \times 16} = 38$ oz. ft.

Ratio of Starting to full load torque $= \frac{120}{38} = 3.2$

With the designed Values design sheet is prepared.

DESIGN SHEET.

H. P. ½ S.R.P.M. 1500 Cycles 50 Poles 4 Volt 230 Amps 3.14

	Stator	Rotor		Stator	Rotor
Outside diameter	5 7/16"	3.471"	Tooth Face	5/16"	.545"
Inside diameter	3.5"	3/4"	Tooth width	9/64	.29"
Length	4.25"	4.5"	Depth below slot	3/8"	5/8"
No. of slots	28	20	Slot factor		
Tooth pitch	.395"	.545	Total cr. section	.0665	.049

Slot number	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Stator Main	53	43	24	-	24	43	53	53	43	34	-	24	43	53
Stator auxi-liary	-	33	48	54-54	48	33	--	--	33	48	54-54	48	33	-

Winding	Main	Auxiliary	Rotor	
Conductor size	19 SuperE	23 SuperE	Bar Section	5/32" x 5/16
Conductor area	.00012569	.00045239	Length	5.47
Amperes per Sq.in.	2500	19800	Material	Copper
Half mean turn	7"	7.45"	Skew bars	--
Conductor in series	960	1080	End ring section	5/16" x 1/4
C w.	.8	.875	Materials	Copper
Weight	2.71	1.165	Dia-outside	2.846

Gap Length .0145" Gap Coefficient 1-14 Pole Pitch 2.75"

Total Flux 1.61 x 10^6 fd 1.637 Flux per pole .256x10^6

	Section	Density	Weight	Core loss
Stator Teeth	2.48	103,000	3.06	12.24
Stator Yoke	2.96	86500	6.5	18/30.24 x 2
Rotor Teeth	4.05	63000		
Rotor Yoke	5.5	46500	—	60.48
Air gap	7.6	33600		

R_{1M}	4.35 at 25°C	R_{1S}	13.25	X_{LM}	13.79	X_L	20.3
R_{2M}	6.60	R_{2S}	10	Z_M	17.6	Z_S	25.7
R_M	10.95	R_o	23.15	K	1.23	X_c	31.82

CHAPTER XII

CONSTRUCTION.

12.1 Materials used.

The materials that were used for the construction of the machine consists of the stator casing cast out of iron, rotor shaft turned out of mild steel. Self aligning bearings were used. Bearing cups are also made of cast iron. The terminal board is of hard rubber. The stampings used were provided with paper insulation. Clamps were used to keep the stator clampings tight in position. For the main winding 19 gauge enamelled wire was used. For the starting winding 23 gauge enamelled wire was used. For the stator slot insulation both leatheroid paper and empire cloth were used. For impregnating the winding about 1/3 gallon of impregnating varnish was also used. Also 1" cotton tape was used to insulate the intercoil connections and overhanging portion of the coil. Soldering lead was also used for soldering the connections. S.W.G. No.2 copper wire was used for the rotor cage.

The various appliances that were used consists of lathe, shaping machine, drilling machine spot welding machine, electric heaters, soldering iro n, gas welding plant, clamping press, set spanner, adjustable spanner and file.

~~The xxxxxxx x~~

12.2. Construction of the Stator.

The punchings had been assembled inside the frame having a key for guiding to avoid the slight overlaps. While compressing the stator stampings by means of a carpenters

STATOR FRAME BEFORE MACHINING

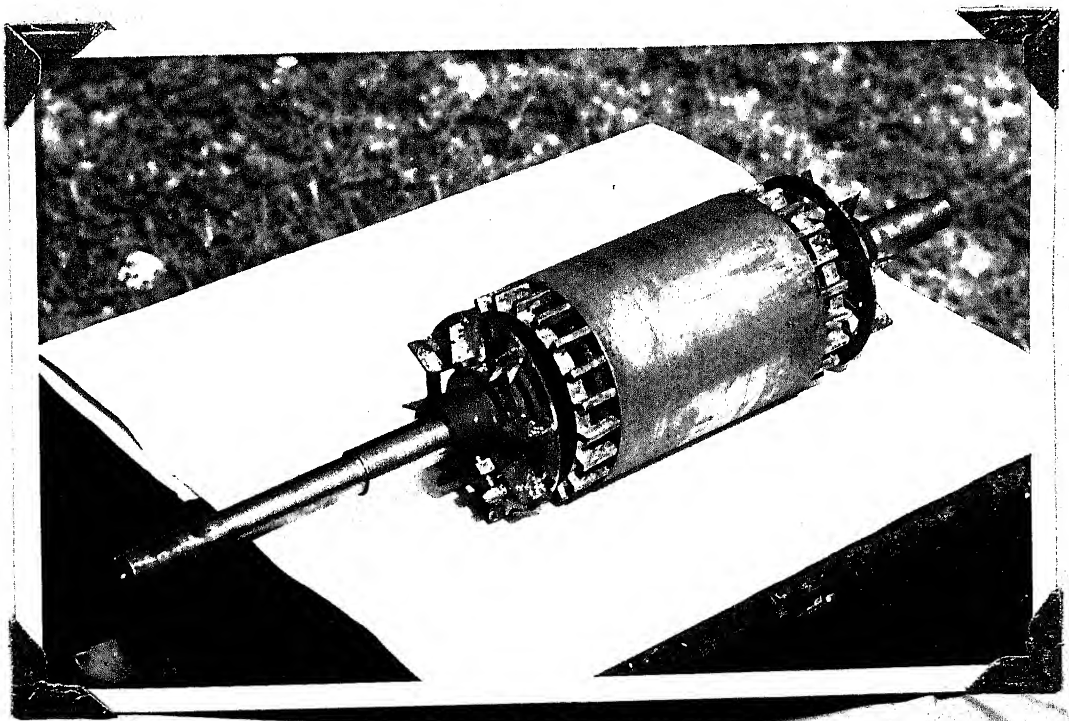
STATOR READY FOR WI

vice some teeth at the ends may bend outward. To avoid this there must be two end plates of about 1/8" thickness. To start with the assembly, one of the above two plates was placed first. Then one by one stampings were placed to a length which would be greater than the necessary core length by about one centimeter. The stampings were then pressed together by means of clamps at three or four places. Channel shaped clamps were made from 1/8" thick metallic strips. A screw arragemnt also was made on one side of the above strip and it was tightened to keep the clamps tight. Afterwards the stator was painted with insulated varnish.

12.3 Construction of the Rotor.

Before stacking the stampings on the shaft it is necessary to provide an end plate to prevent the stampings from fanning out. The end plates were made out of cast iron 1/8" thick.

One of the end plates was slipped over the shaft. Few stampings also were stacked over it and two of the rotor rods were introduced for proper alignment of the holes. Rotor stampings were then stacked one above the other until the length of the stampings was 1 cms. in excess of the desired length. During stacking care was taken to see that there was paper insulation between every pair of stampings. There afterwards these stampings were pressed together by using a carpenter's clamp and then in between the vice until the stampings were well tight. Then they were clamped in position temporarily.



COMPLETED ROTOR

No.8 gauge copper wire is the nearest gauge to suit the hole in the rotor stampings. The rods were made to the correct size before they were inserted into the rotor stampings.

The copper bars were introduced with a light force so that the cage may not be loose in the rotor. The end rings were then placed in position with sufficient clearance from the stampings. All the joints were then properly welded using a gas welding equipment.

The next was taken up after the stator stampings were assembled. These are the following: To determine the diameter of the rotor very accurately to provide the correct air gap the diameter of the stator opeing was measured correct tothe one thousandth of an inch. The diameter of the rotor was pitched upon to provide 29/ 1000 inch gap. The shaft was then mounted on a lathe using a dog. After putting grease the lathe was started and thin layers of metal were removed. Emery paper was applied at the end to get a good finish. After this work steps were taken to fix the bearing. A final finish was given using emery paper to enable the bearing to be driven into position without excessive pressure. The bearing was driven into position by hitting with a wooden hammer uniformly all around. A pipe of suitable diameter was used for this purpose.

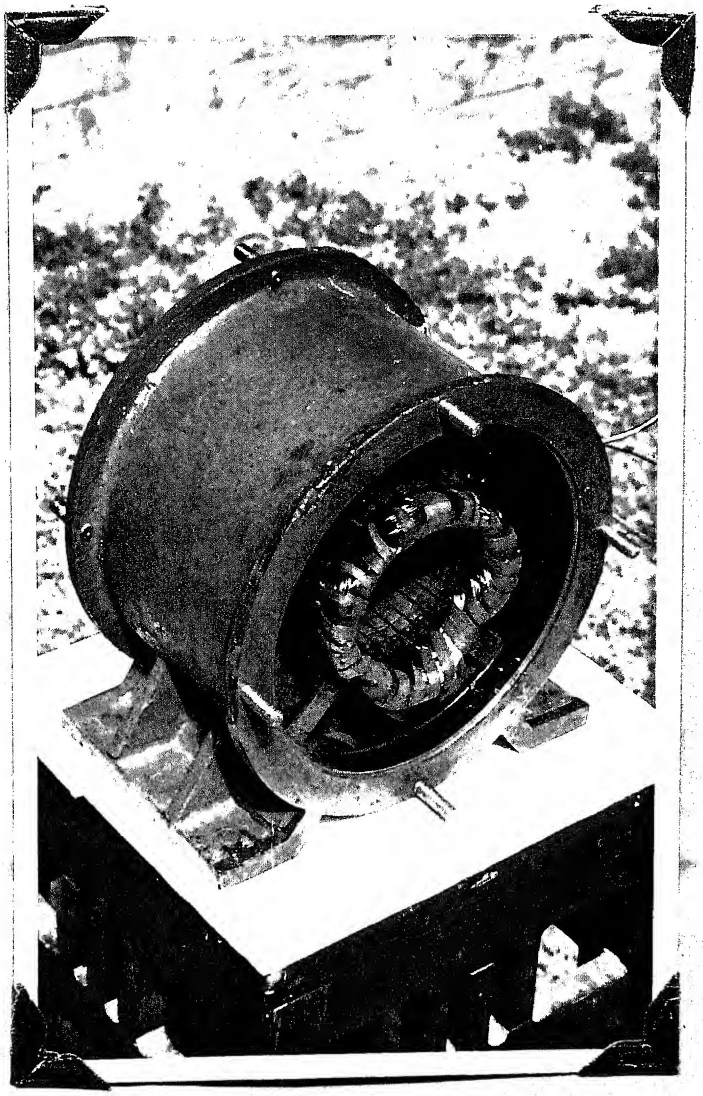
It is necessary to ensure that the rotor is placed symmetrically with respect to the stator. For this after fixing one of the end covers the rotor was inserted into the stator and measurements were taken after fiting that side. The rotor was then taken out and the end cover was

removed. A spacer of correct thickness was provided between the flange and the stator stamping and it was well bolted to prevent that one from flying off while the machine is running. The stampings were then pressed again and a counter sunk bolt was provided for the flange which can slip over the shaft. This flange was kept pressed on the stampings before drilling the depression on the shaft. The rotor was then ready for use.

12.4 Winding.

For the main winding 19 gauge enammelled copper wire was used and for the starting winding 23 gauge enammelled copper wire was used. The coils were hand wound Leatheroid Paper and empire cloth were cut to suit the slot depth. A overhang of not less than 1/2 cms. was provided at each end. The breadth of the leatheroid paper was just sufficient to cover the slot. Extra breadth was provided for the empire cloth for an easy handling and a final finishing out covering the coil completely. The paper was first placed in the slot and then the cloth. The conductors were then inserted one by one. At times itk was found necessary to put a fresh empire cloth insulation when the original one got torn. The main coils were put first. Then the starting coil.

Coil connections were made as per the winding diagram. The number of poles and their polarity were determined by means of a magnetised needle. After ensuring proper connections all the joints were scrapped and soldered. The inter coil connections were well taped and insulated. They



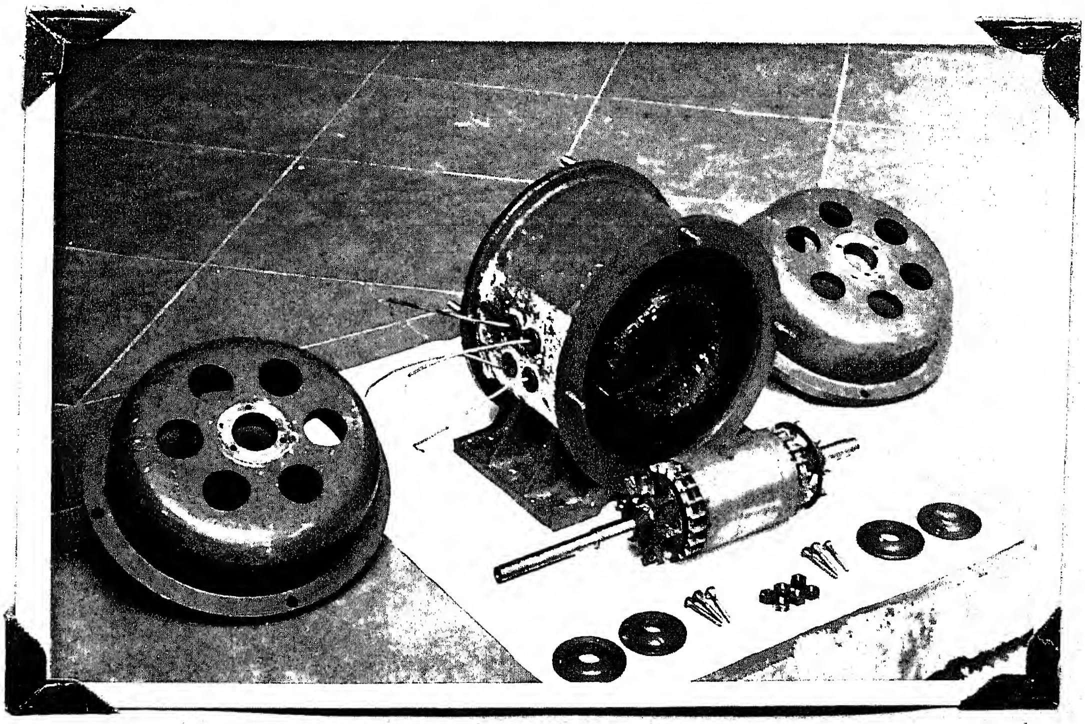
WOUND STATOR

were then soldered to the terminal legs and were screwed on to the position of the terminal box.

The winding was then to be impregnated with insulating varnish. Before proceeding with the work the coils were dried. Two electric heaters were kept on both sides of the winding. A thermometer was inserted into the winding. The insulation resistance was measured. The heaters were then switched on. The temperature was raised to 101°C The insulation resistance was measured at frequent interval. There was a sharp decrease in the resistance initially and then it started increasing. The temperature was maintained for about 2 hours till the insulation resistance became steady at a high value. Care was taken to see that the coil was not over heated. The change in resistance is due to sweating. The moisture in the insulattion collects at the surface initially which slowly gets e vaporated.

In the meantime sufficient quantity of insulating varnish was taken and was thinned. A large pan was also kept ready. After heating for sufficient time the stator with casing was placed in the pan and the varnish was poured over the winding while hot. After the winding was well impregnated it was left in position for drying by itself.

After drying the same was cleaned and now the various parts are ready for assembling. The rotor was taken and the bearings were cleaned with kerosene to remove dust particles sticking on. The bearing cups were filled with grease. The rotor was inserted into the stator and the end shields were fixed. The grease cups were also bolted tight. To



COMPONENT PARTS OF THE MOTOR

enable the machine to be coupled to another one a flange was fixed on to the shaft. The machine was then ready for conducting the tests.

* * *

CHAPTER XIII.

Tests.

13.1. A Commercial Test.

A Commercial Test on a motor is a simplified test that usually gives most of the desired information about the Motor. It is a test such as is taken by manufacturers on each production motor before shipment. It is the sort of Test that should be taken and recorded by repair shops. Such a test consists of the following:

a. No-load Readings:

Allow the Motor to run at no load for a sufficient time for the bearings to warm up (10 min to ½ hr. or more will be satisfactory). Then measure the amperes, watts, and r.p.m. at rated voltage. While the motor is running, it is advisable to check for unusual or abnormal noises. Such as rubbing insulation, or dirt in the air-gap: also it should be observed whether or not the rotor "floats" within the limits of the end play. That is does the rotor seem to seek a position in the middle of the end play, or does it crowd excessively against one bearing or the other. This condition will indicate whether or not the rotor core is properly located with respect to the Stator core.

(b) Full load Readings:

Holding the output at rated full load torque, measure

the amperes and watts input, also the r.p.m.

(c) Locked-rotor and Breakdown Torques.

The locked-rotor and breakdown torques should be taken. The method of taking these is described in more detail in Art ? c.c.

12. 2. A Complete Engineering Test:

More complete information about a Motor is often desired, and for this reason, there are many other tests that may be taken as described in the following paragraphs:

(a) Cold Resistances:

The cold resistances of each of the windings should be carefully measured and the room temperature recorded. For record purposes, the resistances so measured should all be corrected to a standard room temperature of 25°c by the following formula:

$$R_{25^\circ C} = \frac{260}{235 + t} \; rt$$

Where $R\ 25^\circ C$ = Resistance at 25°C

rt = resistance at temperature t.

(b) Locked rotor Readings:

Locked rotor readings on fractional horse power motors can be taken at approximately rated voltage. The Watt meter, Volt-meter, and ammeter, connected should be read simultaneously

and the readings should be taken within 4 or 5 sec of the time of application of the voltage; for the windings heat rapidly and this heating effects the accuracy of the readings. It is not necessary that the line voltage be exactly rated voltage, but it should be set somewhere near the correct value, and all the three meters must be read simultaneously as soon as the needles have settled down to a steady reading. Immediately after the reading, the primary resistance should be measured and recorded. With split phase motors, a separate set of locked rotor readings of both main and auxiliary windings should be taken; when the readings on one winding are being taken, the other winding need not be excited.

(c) Break down Torque:

The breakdown torque of an Induction motor is the maximum torque that the motor will develop at rated voltage and frequency, without an abrupt drop in speed. Break down torque is taken by increasing the torque load on the motor until the motor "pulls out" i.e. drops off in speed sharply or stops entirely. Care must be taken to increase the load slowly enough so that the inertia effectd do not increase the apparent breakdown torque.

(d) Switching and Pull up Torques:

Switching torque is a term used in recent A.S.A. and A.I.E.E. standards to denote the minimum external torque developed by a motor as it accelerates through the switch operating speed. This torque is an essential characteristic of any

single phase motor with an automatic switching mechanism such as a starting switch or short-circuiter. Pull-up torque of an a.c. Motor is the minimum external torque developed by the motor during the acceleration of the motor from rest to speed at which break down torque occurs.

For taking the Switching torque of a capacitor-start motor, start with a light load, increasing the load gradually by means of the brake arm until the motor "breaks down" i.e. falls off in speed abruptly into the starting connection. Since the torque on the starting connection is greater than the torque on the running connection, the rotor will accelrate and the switch will again operate; if the brake ~~down x~~ arm were adjusted for the breakdown torque of the motor, the motor would "pump" i.e. would pass alternately from starting to running connections, and vice-versa. If the torque of the brake arm is slowly reduced by loosening the wing nuts, the motor eventually will cease pumping and stay on the running connection. This highest value of torque at which the motor will cease pumping and stayon the running connection is the switching torque.

(e) <u>Locked-rotor Torque and Current.</u>

The locked rotor torque of a fractional horse power motor varies with rotor position. One method of measuring this torque is to obtain seven or eight square pieces of wood 3/16 in thick by approximately 2 in square. The whole pile can be set on the scales and the brake arm set on top of the complete

pile. The tare is determined with the blocks in position. The locked rotor torque is measured with the complete set of blocks; then one block is removed but left on the scales and the locked-rotor torque measured in the second position; then the second block is removed from the pile but left on the scales and the torque again noted; and so on, repeating until the torque has been taken in all the positions. When taking these tests, the voltage should be held at rated line Voltage, and the current should be noted with an ammeter and recorded. The readings should be taken as quickly as possible to prevent overheating of the Motor. In the case of Capacitor start or two-value capacitor Motors, the capacitor Volts should also be measured. Only the maximum and minimum values of locked-rotor torque need be recorded. An alternative method of testing is to turn the motor over onto the round part of the frame, rolling the Motor slowly through a small angle and noting the maximum and minimum values of the locked rotor torque. If this procedure is followed, the Motor should be rolled away from the scales so that the bearing friction is subtracted from the developed torque, instead of added to it, as would be the case if the motor were rolled toward the scales.

(f) <u>Running Saturation:</u>

The purpose of the running Saturation test is to aid in the segregation of losses; this test permits the separate determination of the friction and windage, also the core losses of the Motor. With the Motor running at no load take a series of readings of Volts, amperes and Watts, Starting at 130 per

per cent of rated voltage and varying the voltage downward until the motor loses speed rapidly or until the current starts to increase.

(g) Brake Test

The purpose of the brake test is to determine the variation of power factor, efficiency, Watts, amperes, and r.p.m. with changes in load. Starting at as near break down torque as possible, hold the load constant at one value of torque, and read the Watts input, amperes, r.p.m. Slip by torque, and read the Watts in put, amperes, r.p.m. Slip by stroboscopic means, and torque. Repeat these readings for different settings of torque down to no load. The horse power output is computed. The readings may all be plotted against torque oragainst horse power output.

(h) Full-load Saturation:

The purpose of the full load saturation test is to determine how the full-load performance of the Motor varies with a change in line voltage. Holding the output torque of the Motor constant at full load, vary the applied Voltage: readings are to be taken at seven voltages, viz. one reading at ratd line voltage, three in steps of 10 per cent each above line voltage, and three more in steps of 10 per cent each above line voltage, and three more in steps of 10 per cent each below line voltage. These readings should be plotted against Voltage.

(i) Temperature Run.

Usually it is not practicable or feasible to take a

a temperature run at full load by means of a brake arm, for if the motor is of any substantid horse power rating, the brake arm will get too hot, and even if it does not continuous attention is required for 2 or 3 hr. If a dynamometer is available, the output torque can be held constant at rated value. If not, the motor can be belted up to a generation or some other load, and the load can be adjusted until the Watts in put shows that the Motor is delivering full load output. Thermometers (or thermocouples) should be placed on the hottest accessible partsof the frame and the primary copper, and read during the run, they should also be placed on the Secondary copper and Secondary iron after the run is shut down. Record Volts, Watts, amperes, slip, and temperature every 15 min. throughout the Test-unless the Motor has an intermittent rating- until the temperatures become constant. The additional thermometers should be preheated to the expected rise before the Motor is shut down and then immediately placed in position. The hot resistance of the winding should be recorded immediately after shutdown.

Unless it is known that the motor operates on iron, the temperature run should be taken with the motor mounted on wood. Also care should be taken to shield the motor from drafts, particularly if it is an enclosed motor.

(j) Speed-torque Tests:

Dynamometers are necessary for taking a complete Speed torque test of a single-phase induction motor; neither the

prony-brake nor the cord and puller method is satisfactory for obtaining points between the breakdown and zero speeds. For most satisfactory results the field of the dynamometer should be connected to a fixed voltage d.c. bus in series with a field rheostat.; the armature of the dynamometer should be connected to a separate d.c. bus, the voltage of which is preferably adjustable. The procedure for obtaining a point on the speed-torque curve is as follows: the dynamometer is started as a motor with full field and at a low armature voltage to give a low operating speed: the a.c. line voltage is adjusted to a value slightly above rated value, then the switch is closed to apply the voltage to the motor being tested, motor voltage is quickly adjusted to rated value, and the torque and speed are both simultaneously noted and recorded. Generally speaking, the less the speed changes when the a.c. power is applied, the easier it is to obtain good readings. In order to avoid over heating the motor and thereby obtaining false readings, it is essential to apply the a.c. voltage to the motor for not more than 4 or 5 sec for each reading, and further, the motor must be allowed to cool between readings.

13.3 Conclusion.

The actual results obtained from the tests agree closely with the results got from the design data. This can be seen from the following tabulation.

No.	Details - Constants	From Design datas	From Test results
1.	Main Winding resistance RlM	4.35 ohms	6.13 ohms
2.	Start winding resistance RlS	13.25 ohms	14.6 ohms.
3.	Rotor Resistance in terms of Winding R 2	6.6 ohms	10 ohms
4.	Stator leakage reactance Xl	12.27 ohms	12 ohms
5.	Sec. reactance in terms of Stator	2.0 ohms	4.35 ohms
6.	Magnetising reactance Xm	169. ohms	175. ohms
7.	$X_o = X_m + \frac{X_1 + X_2}{2}$	175.9	183.2
8.	X = Ideal S.C. reactance $x_1 + \frac{X_m \ x_2}{x_m + x_2}$	13.7	16.8
9.	Kn = leakage factor	.96	.94
10.	No load loss	96	80
11.	Friction and Windage loss	12.6	15.9
12.	Iron Loss	60	20
13.	No load current	2.2	2.3
14.	Full load current	3.43	3.5
15	Full load p.f	69.8%	64%
16.	Full load input	547.25wats	575
17.	Full load efficiency	68%	64%
18.	Full load Torque	29.803 ft	31 oz ft
19.	Starting Torque	100 oz ft	700 oz ft
20	S.C. current for main phase	13.08 amp	10.25
21.	S.C. current for start pahse	8.95	6.9
22.	Total S.C.(starting current)	23 amps	13.5
23.	Temperature rise	40^o C	
	By thermometer		37.5^o C
	By Resistance Measurement		44 C

No Load Saturation Test Results.

Voltage	Current	Watts.	Cu. Loss	Iron, Friction and Windage
270	3.12	140	103	37
260	2.92	112	90.5	39.5
250	2.74	110.5	80.0	30.5
240	2.55	99.5	69.0	30.5
230	2.35	90.	58.5	31.5
220	2.22	80.	52.0	28.0
210	2.09	60.	46.5	13.5
200	1.95	49	40.5	8.5
190	1.8	42	34.5	7.7
180	1.6	47	27.2	19.8
170	1.56	40	26.1	13.9
160	1.45	36	22.4	13.6
150	1.34	30	19.2	10.8
140	1.24	25	16.4	8.6
130	1.14	20	13.85	6.15
120	1.05	19	13.4	5.6
110	0.95	15	9.6	5.4

Load Test

current	Watt	power factor	T_1-T_2	Speed	Slip	Torque oz-ft.	B.H.P	Output per-cent-age.	Effi-ciency percen tage.
2.34	82	.152	0	1496	.266	0	0	0	0
26	290	.328	2.8	1470	2.0	11.1	0.196	39.2	50
2.88	360	.400	4.35	1450	3.33	13.25	.3	60	61
3.5	505	.629	6.6	1419	5.4	26.2	.445	89	66
3.05	395	.562	5.25	1420	5.33	20.9	.345	70.8	67
3.3	466	.615	6.4	1400	6.66	26.5	.425	85.0	68
3.25	455	.61	6.0	1405	6.33	25.2	.4	80	65.5
3.32	472	.62	6.55	1400	6.66	26.5	.435	87	68.8
3.7	552	0.65	7.75	1375	8.35	33.2	.505	101.1	682.
3.2	550	.75	7.85	1380	8.0	3.3	.515	103	70
3.8	570	.655	7.1	1375	8.35	33.2	.465	93	61
4.0	630	.682	8.3	1355	9.7	38.6	.535	107	63.5
3.9	605	.675	8.4	1365	9.0	35.8	.545	109	67.2
4.0	620	.675	8.35	1358	9.45	37.6	.54	108	65
4.2	665	.690	9.1	1330	11.3	45	.575	115	64.5
5.47	910	.725	11.5	1340	11.5	47.5	.7	140	58

Heat Run Test

Volt	Current	T1	T2	Watts	Tem-pera-ture.	Speed	Time	Room tempera-ture.
230	3.5	12.1	5.5	490	36.	1410	10.0	28
230	3.5	12.5	5.4	510	49.5	49.5	10.3	28
230	3.5	12.5	5.5	520	570	1395	11.0	29
230	3.5	11.0	4.2	515	59.0	1395	11.30	29
230	3.75	12.5	4.2	575	61.5	1400	12.00	29
230	3.4	12.0	5.5	500	63.5	1410	12.30	29
230	3.6	13.	5.5	545	66	1390	1	30
230	3.65	13	5.0	555	67.0	1395	1.30	30
230	3.2	12	5.2	485	67.2	1410	2.0	30
230	4.0	12	5.2	600	67.2	1390	2.30	30
230	3.7	11	4.0	550	67.5	1395	3.0	30
230	3.6	12	2.0	520	67.5	1400	3.30	30
230	3.5	11	3.5	520	67.5	1400	4.00	30

B I B L I O G R A P H Y.

BOOKS.

B.1. Electric Machinery Vol. II A.C. Machines by Garik and Whipple.

B.2. Induction Motor by Vickers.

B.3. The Performance and Design of A.C. Machines by Say.

B.4. Electrical Machine Design by Alexander Gray.

History of the Induction Motor.

B. 1. The Induction Motor by Herbert Vickers Pages 1 - 6.

B. 2. The Performance and Design of Alternating Current Machines by M.G. Say Page 249.

B. 3. Electric Machinery Vol. II. A.C. Machines by Garik and Whipple.

Rotating Magnetic Field Theory.

J. 1. Rotating Magnetic Field Theory of A.C. Motors by Hansen. A.I.E.E. Journal Page 340 Vol. XLIV 1925.

B. 1. The Induction Motor by Vickers Page 105.

B. 2. Electric Machinery Vol. II. A.C. Machines by Garik and Whipple.

B. 3. The Performance and Design of A.C. Machines by Say.

B. 4. Alternating Current Machines by A.F. Puchstein and T.C. Lloyd.

B. 5. Electrical Machine Design by Alexander Gray.

Cross Field Theory.

J. 1. The Cross Field Theory of Capacitor Motor by Puchstein and Lloyd. A.I.E.E. Vol. 60. 1941 Page 58.

J. 2. " On the Equivalence of the Two Theories of the Single Phase Induction Motor" A.I.E.E. 1921 by Karapetoff.

J. 3. Single Phase Motor Theory C.T. Button A.I.E.E. Page 40-151. 1940.

J. 4. A Physical conception of Single Phase Motor Operation R. Beach. Electrical Engineering 1944 Page 254.

Correlation of the Cross Field and Revolving Field Concepts:

J. 1. The Cross Field Theory of Alternating Current Machines by H.R. West. A.I.E.E. Journal 1926. Pages 466-74.

J. 2. Revolving Field Theory of Capacitor Motor W.J. Morrill 1929.

J. 3. A suggested Rotor Flux locus concept of Single Phase Induction Motor operation. C.T. Button 1937 Pages 331-32.

J. 4. Single Phase Motor Torque Pulsations. A.L. Kimball and P.L. Alger A.I.E.E. 1924. Pages 730-739.

J. 5. Single Phase Motor Theory. Page 507 J.I.E.E. Vol.60 1941.

Symmetrical Components as applied to Single Phase Induction Motor.

J. 1. Analysis of Unsymmetrical Machines by W.V. Lyon, Charles, Kingsley, A.I.E.E. Vol.55, 1936 Pages 471-6.

J. 2. The Revolving Field Theory of the Capacitor Motor W.J. Morril A.I.E.E. Vol. 48, 1929 Pages 614-32.

J. 3. The Apparent Impedance Method of Calculating Single Phase Motor Performance. W.J. Morril, A.I.E.E. Vol.60, 1941 Pages 1037-41.

J. 4. Electric Circuits-- Theory and Application O.G.C. Dahl. Pages 80-9, 138-44.

J. 4. Induction Motor Core Losses P.L. Alger, R.Ekergian, A.I.E.E. Journal Vol.39, Pages 906-20. 1920.

N O I S E.

B. 1. Electrical Machine Design. Alexander Grey. P.337.

J. 1. Quiet Induction Motors L.E. Hildebrand A.I.E.E. Vol.49 July 1930 Page 848.

J. 2. Study of Noises in Electrical Apparatus T.Spooner and J.P. Foltz. A.I.E.E. Vol. 48, Page 747.

J. 3. Cause and Elimination of Noise in Small Motors by
W.R. Appleman A.I.E.E. Vol.56 '37 Page 1359.

J. 4. The Magnetic Noise of Polyphase Induction Motors by
P.L. Alger, A.I.E.E. Vol.73, P. 118, 1954.

J. 5. Reduction of Noise produced by Small and Medium 2 pole
turbine generators by A.J. Wood A.I.E.E. Vol.73 P.36.

J. 6. Harmonic Theory of Noise in Induction Motors by W.T.
Morrill A.I.E.E. Vol.59. P.474, 1940.

J. 7. Causes of Noise in Induction Motors by R. Wiggen bach.
Brown Boveri. Rev. Vol.20. October 1933 P.123.

J. 8. Performance of Noise Meters in terms of the primary
Standard by Churcher, and King. A.I.E.E.'37 V.81 P.57.

J. 9. The Analysis and Measurement of Noise emitted by Ele-
ctrical Machinery by King and Churcher. J.I.E.E. Vol.68.
1930 P.97.

Power Losses in Induction Machines.

J. 1. Squirrel Cage Induction Motor Core Losses by T.Spooner
A.I.E.E. Vol. 44 P.155. 1925.

J. 2. Power Losses in Induction Machines by Paula Harbutoviskih
A.I.E.E. Vol.53 P.1466, 1934.

J. 3. Physical concept of Stray Load loss in Induction Machines
by S.S.L. Chang A.I.E.E. Vol.73 1954 Page 10.

J. 4. Surface Iron Losses with reference to terminated materials
by T. Spooner and I.F. Kinnard, A.I.E.E. Vol.43, 1924
Page 262.

J. 5. A method for separating No load Losses in Electrical Machinery by Carl. J. Fechhe Tner A.I.E.E. Vol.XXXIX Page 291, 1920.

J. 6. Tooth frequency Iron Losses in Slip ring Induction Motors D.B. Hoseason. The Electrician Vol.91, 1923 Page 240.

J. 7. Tooth Pulsations in Rotating Machines by T. Spooner A.I.E.E. Volume 43, 1924 P.252.

J. 8. Induction Motor Core Losses P.L. Alger and R. Eksergian. A.I.E.E. Vol. 39, P. 906, 1920.

J. 9. Induction Motor Load Losses by H.G. Reist and A.E. Averett. A.I.E.E. Vol. 32, 1913, Page 423.

J. 10. The Unsymmetrical Hystersis loop T.D. Hull, A.I.E.E. Trans. V. 34. Page 2693 1915.

J. 11. I.E.E. Journal Vol. 37, 1906 Page 125 by Dr.V.M.Thornton.

<u>Temperature Rise.</u>

B. 1. Electrical Engineering Design Manual by Say and Smith.

B. 2. Design and Performance of A.C. Machinery by M.G. Say Page 247.

B. 3. B.S.S. 169- 1925.

B. 4. B.S.S. 2361- 1955.

B. 5. Fractional Horse Power Electric Motors- Veinott Page 461 and 473.

J. 1. Temperature differential in Single end Ventilated Machines by G.P. Potter.

A.I.E.E. Trans. Vol.71, 1952.

J. 2. The thermal time constants of Dynamo Electric Machines by A.E. Kennely. A.I.E.E. Vol. XLIV P.137 '25.

J. 3. Measurement of Temperature in General Purpose Squirrel Cage Induction Motors by S.P. Potter. A.I.E.E. Vol.58 p. 468, 1939.

J. 4. Determination of Temperature Rise of Induction Motors Vol.58, P.459, 1939, by E.R. Summers.

J. 5. An equivalent Thermal circuit for non Ventilated Induction Motors by Robert L. Rothik, A.I.E.E. Vol.73. Page 1604, 1954.

J. 6. Accuracy of Temperature Measurement on Coil Surface by Martin and Bernard, A.I.E.E. '47. Vol.66, P.536.

J. 7. Cooling of Electrical Machines by Moseason A.I.E.E. Vol.69. '31 P121.

J. 8. Application of propeller fans to the cooling of Electrical Machines by R. Poole. A.I.E.E. Vol.77, '35. p. 293.

J. 9. Measurement of Temperature in General Purpose Squirrel Cage Induction Motors by Potter A.I.E.E. Vol.56, 1937, Page 253.

Rating of Induction Motors.

J. 1. Electrical World August 1926 Paper by C.L. Collens.

J. 2. American Standard, for Rotating Electrical Machinery, A.S.A. Publication C.50, approved January 6, 1936.

J. 3. Determination of Temperature Rise of Induction Motors, E.R. Sammers. A.I.E.E. Transaction Vol.58, 1939, Page 459.

J. 4. Loading Transformers by Temperature V.M. Montsinger A.I.E.E. Journal Vol. 49, 1930, Page 293.

J. 5. Temperature and Life characteristics of class A insulation J.J. Smith and J.A. Scott, A.I.E.E. Trans. Vol.58 1939, Page 435.

J. 6. British Standard specification for the Electrical Performance of Industrial Electric Motors and Generators with Class A Insulation.

J. 7. Measurement of Temperature in General Purpose Squirrel Cage Induction Motors by C.P. Potter A.I.E.E. Vol.58. P.468, 1939.

J. 8. Determination of Temperature Rise of Induction Motors Vol.58, P.459, 1939 by E.R. Summers.

J. 9. The thermal time constants of Dynamo Electric Machines by A.E. Kennely. A.I.E.E. Vol. XLIV Page 137, 1925.

J. 10. Insulation of Electrical Apparatus by Willis Jackson.

Design.

J. 1. An Experimental Study of the Induction Machine end turn leakage reactance by Barnes. A.I.E.E. Vol. 70. 1951 Page 671.

J. 2. The Design of Induction Motors with Special reference to Magnetic Leakage by Adams. A.I.E.E. Vol.25, 1905 Page 649.

J. 3. Teaching of the Principles of Electrical Machine Design by Carr. J.I.E.E. Vol.94, 1947. P.443.

J. 4. The Magnetic field of the Dynamo Electric Machines by Carter, J.I.E.E. Vol.64, 1926, P.1115.

J. 5. Calculation of South Well's Relaxation Method J.I.E.E. Vol.92, 1945 Page 522.

J. 6. Saturation effect of leakage Reactance by Lloyd A.I.E.E. Vol.68, 1949, Page 1144.

J. 7. Electric Machinery Vol. II. A.C. Machines by L. Garik.

J. 8. Nature of Polyphase Induction Motor by P.L.Alger.

Construction.

J. 1. Recent Progress in Induction Motor Construction by Carr J.I.E.E. Vol.78, 1936 Page 383.

J. 2. Manufacture of Small Electric Motors Brown Boveri Review '49. Page 160.

J. 3. Design of large Electrical Machine frames by Ralph Moore; Metropolitan Vickers Gazette 1935. Volume 15, Page 420.

J. 4. Shaft current in Electrical Machines by Alger and Samson. A.I.E.E. Vol.43. 1924 Page 235.

T e s t s.

T. 1. Measurement of Stray load loss in Induction Motors by Ware. A.I.E.E. Vol.64 '45 Page194.

T. 2. Tooth Pulsations in Rotating Machines by T.Spooner A.I.E.E. Vol.43, 1924 Page 252.

T. 3. Measurement of Noise from Small Motors by Veinott. A.I.E.E. Vol.53. 1934 Page 1624.

T. 4. Efficiency Tests of Induction Machines by Leader and Philips A.I.E.E. Vol.53. 1934 Page 1628.

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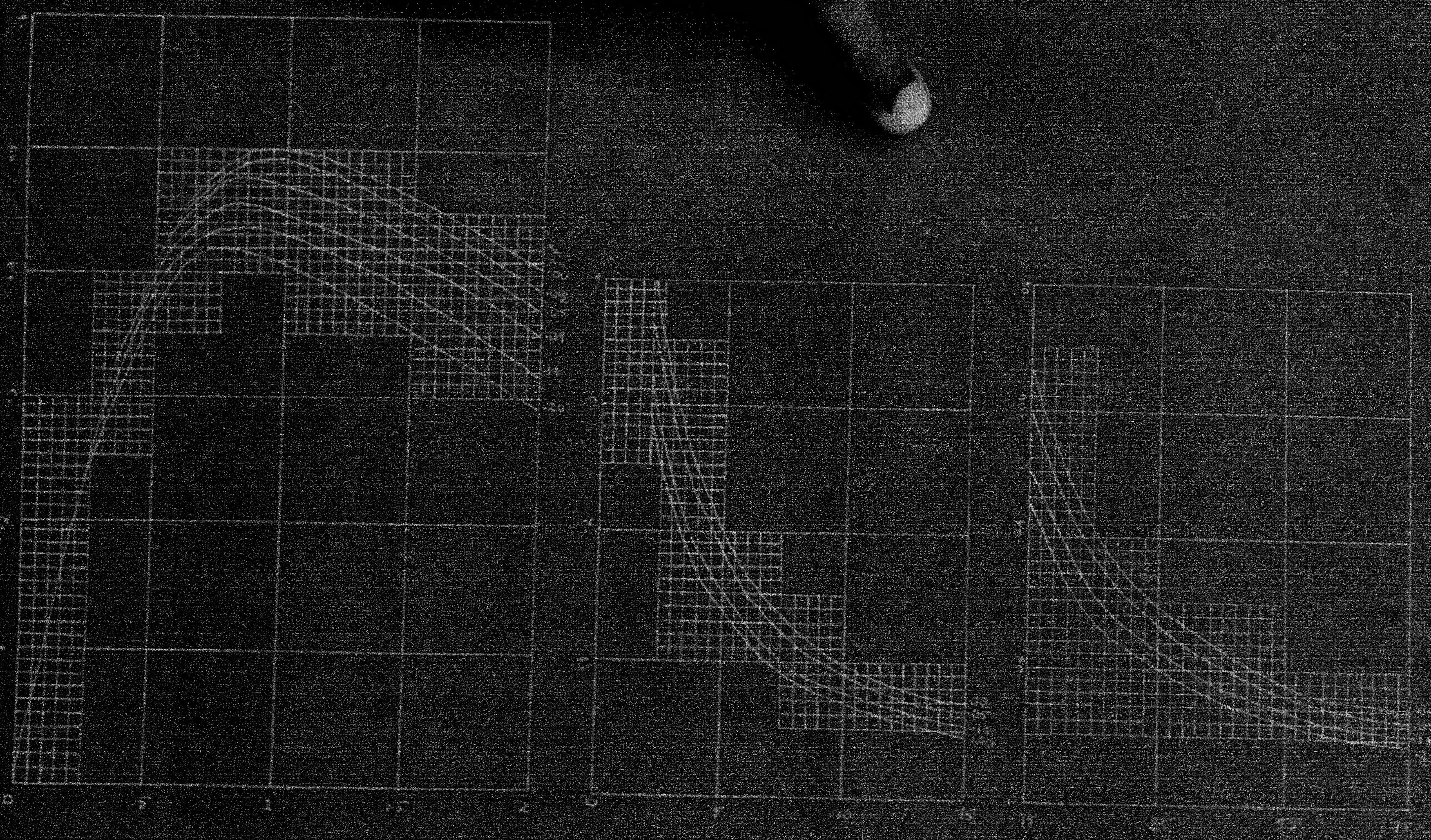
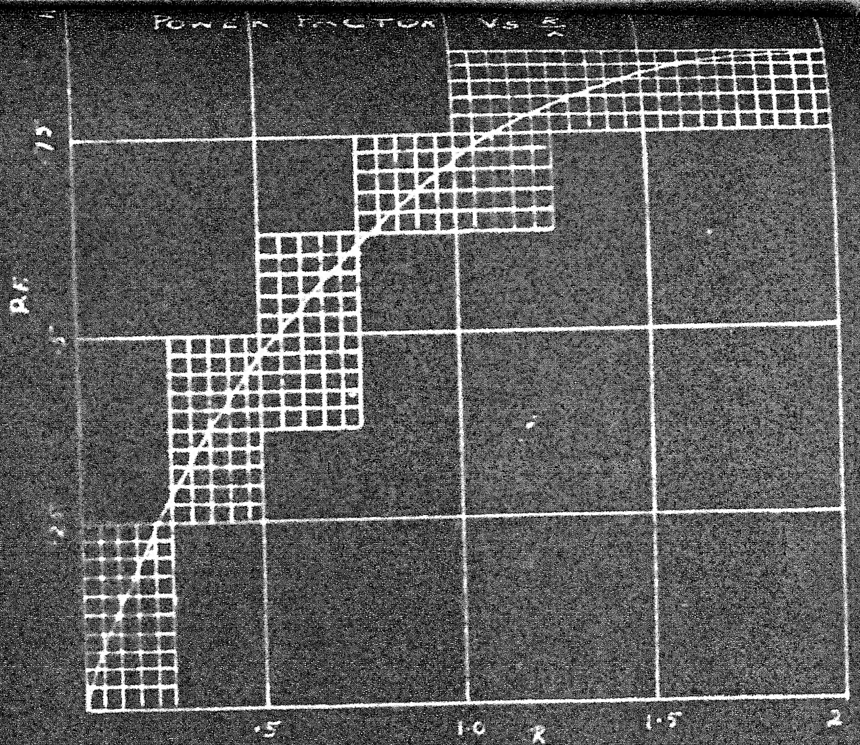
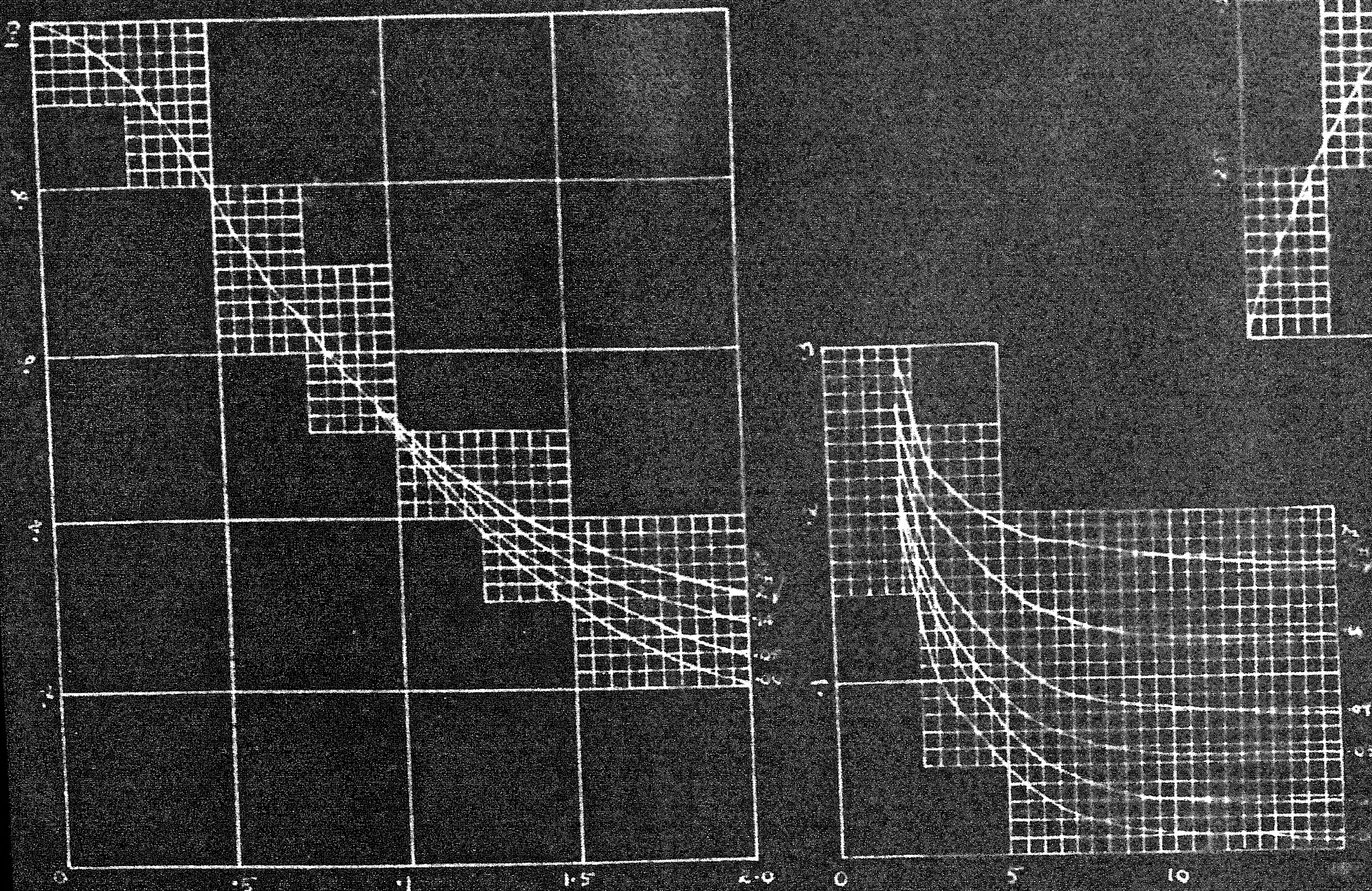
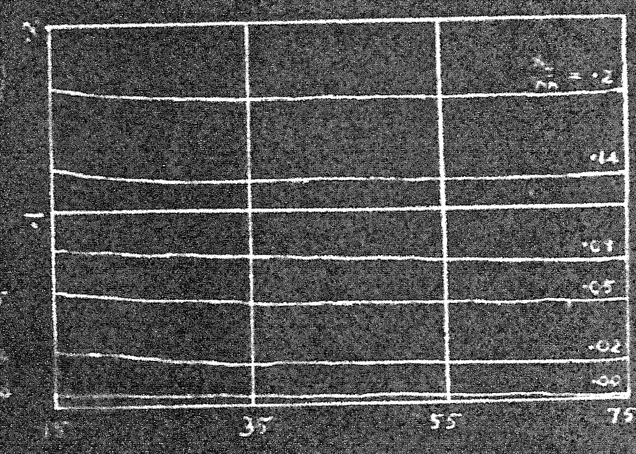
BOOK II



APPARENT RESISTANCE CHART $\frac{x_m s}{K_2}$ OR $\frac{x_m(2-s)}{K_2}$



FIG

APPARENT REACTANCE CHART

FIG

CT. SCALE 1 C.M = 1 AMP.
P.F = COS θ
OUT PUT = 230 x PQ. IN WATTS.
IN PUT = 230 x PR.
EFF: = $\frac{PQ}{PR}$
SLIP $s = 1-\sqrt{1-\ell}$
TORQUE = $\frac{230 \times PQ \times 33000 \times 16}{746 \times 1500 (1-s)}$ IN OZ. FT

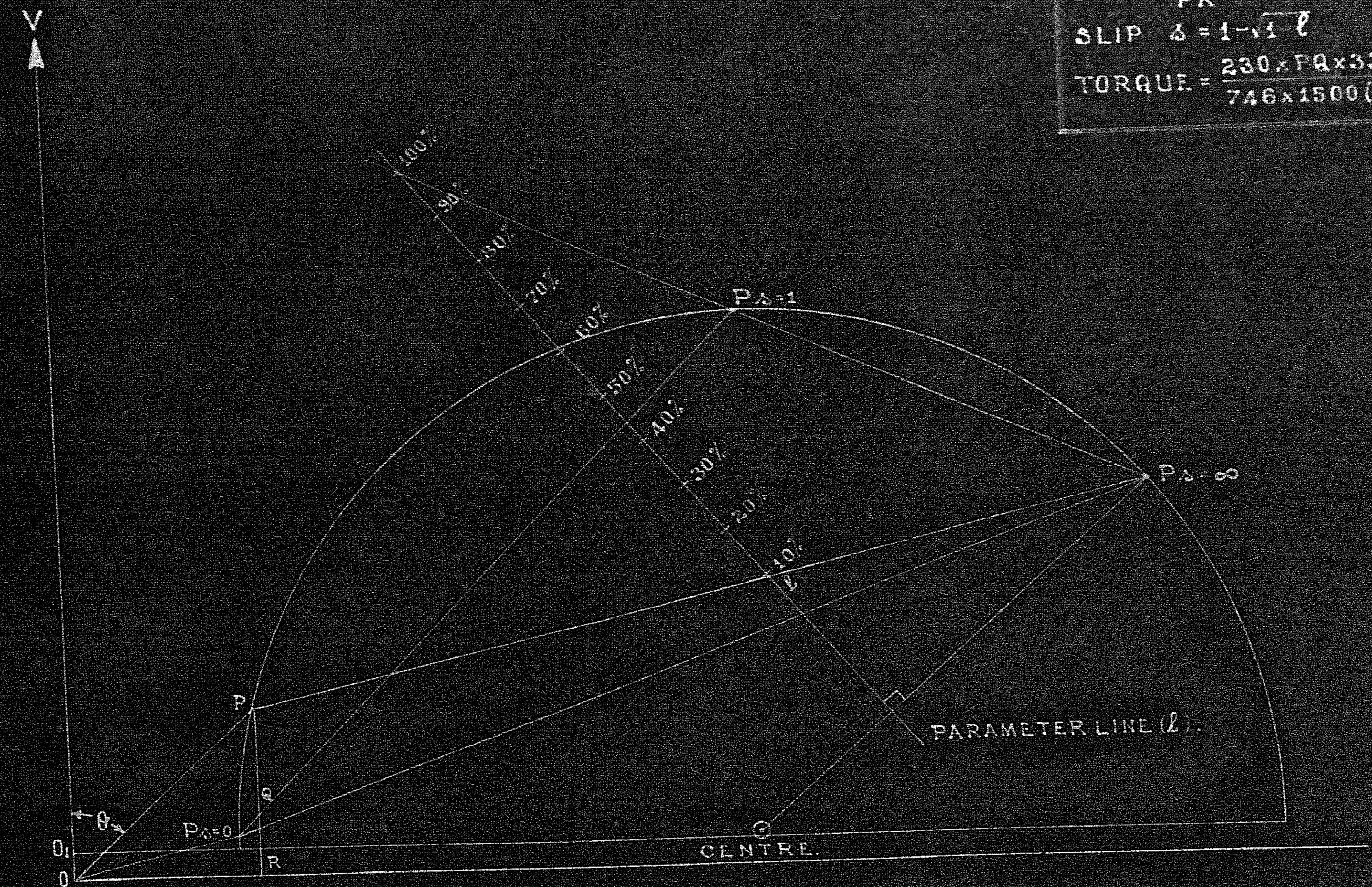


FIG. Nº .